## LECTURE 6

## MODELING EARTHQUAKES

AS TSUNAMI SOURCES

## PRINCIPLES of HYDRODYNAMIC SIMULATIONS

## CLASSICAL APPROACH

1. Obtain model of Earthquake Rupture
2. Compute Static Deformation of Ocean Bottom
3. Use as Initial Conditions of

Vertical Surface Displacement with Zero Initial Velocity
4. Run Hydrodynamic Model (e.g., MOST)
5. Propagate, up to and including

INUNDATION of Receiving Shore


## GENERIC EARTHQUAKE DISLOCATION

Involves MANY parameters

## FIRST STEP

## - Position a point force $\mathbf{F}$ in an infinite homogeneous elastic medium


$\rightarrow \quad$ Obtain the Dynamic displacement field of the deformation

Then for a point force $X_{0}(\mathrm{t})$ in the $x_{j}$-direction at the origin, we have

$$
\begin{align*}
u_{i}(\mathbf{x}, t)= & X_{0} * G_{i j} \quad \text { (in the notation of Chapter 3) } \\
= & \frac{1}{4 \pi \rho}\left(3 \gamma_{i} \gamma_{j}-\delta_{i j}\right) \frac{1}{r^{3}} \int_{r / \alpha}^{r / \beta} \tau X_{0}(t-\tau) d \tau \\
& +\frac{1}{4 \pi \rho \alpha^{2}} \gamma_{i} \gamma_{j} \frac{1}{r} X_{0}\left(t-\frac{r}{\alpha}\right) \\
& -\frac{1}{4 \pi \rho \beta^{2}}\left(\gamma_{i} \gamma_{j}-\delta_{i j}\right) \frac{1}{r} X_{0}\left(t-\frac{r}{\beta}\right) . \tag{4.23}
\end{align*}
$$

[Aki and Richards, 1980; p. 73, Eqn. (4.23)]

- The STATIC displacement is simply obtained by putting $t \rightarrow \infty$.
[This expression is known as the Somigliana Tensor]


## SECOND STEP

- Replace Single Force by Double-Couple

$\rightarrow$ Simply use Somigliana's tensor as a Green's function and take appropriate derivatives.
$\rightarrow$ Note that these are the $P$ and $S$ waves of the near [and far] field[s].

NEAR FIELD
NEAR FIELD
NEAR FIELD
[Far Field]

$$
\begin{aligned}
M_{p q} * G_{n p, q}= & \left(\frac{15 \gamma_{n} \gamma_{p} \gamma_{q}-3 \gamma_{n} \delta_{p q}-3 \gamma_{p} \delta_{n q}-3 \gamma_{q} \delta_{n p}}{4 \pi \rho}\right) \frac{1}{r^{4}} \int_{r / \alpha}^{r / \beta} \tau M_{p q}(t-\tau) d \tau \\
& +\left(\frac{6 \gamma_{n} \gamma_{p} \gamma_{q}-\gamma_{n} \delta_{p q}-\gamma_{p} \delta_{n q}-\gamma_{q} \delta_{n p}}{4 \pi \rho \alpha^{2}}\right) \frac{1}{r^{2}} M_{p q}\left(t-\frac{r}{\alpha}\right) \\
& -\left(\frac{6 \gamma_{n} \gamma_{p} \gamma_{q}-\gamma_{n} \delta_{p q}-\gamma_{p} \delta_{n q}-2 \gamma_{q} \delta_{n p}}{4 \pi \rho \beta^{2}}\right) \frac{1}{r^{2}} M_{p q}\left(t-\frac{r}{\beta}\right) \\
& +\frac{\gamma_{n} \gamma_{p} \gamma_{q}}{4 \pi \rho \alpha^{3}} \frac{1}{r} \dot{M}_{p q}\left(t-\frac{r}{\alpha}\right) \\
& -\left(\frac{\gamma_{n} \gamma_{p}-\delta_{n p}}{4 \pi \rho \beta^{3}}\right) \gamma_{q} \frac{1}{r} \dot{M}_{p q}\left(t-\frac{r}{\beta}\right)
\end{aligned}
$$

[Aki and Richards, 1980; p. 79; Eqn. (4.29)]

## THIRD STEP

- Include effect of free surface
(Combine with "reflection" of equivalent P and S waves)


Fig. 2.6-6 Geometry for a $P$ wave in a halfspace incident upon a free surface. $A_{1}, A_{2}$, and $B_{2}$ are the amplitudes of the incident $P$, reflected $P$, and reflected $S V$ waves.
[Stein and Wysession, 2002]

- Integrate over finite area of faulting


Fig. 1. Fault geometry and coordinate system.

The problem has an analytical solution TWO equivalent algorithms

Mansinha and Smylie [1971]
Okada [1985]
Only difference: Okada allows for tensile crack
(non-double-couple solution).

# STATIC DEFORMATION OF OCEAN BOTTOM 

Straightforward, if somewhat arcane analytical formulæ
[Mansinha and Smylie, 1971; Okada, 1985]
yoshimitsu okada
(1) Displacements

For strike-slip

$$
\left\{\begin{array}{l}
u_{x}=-\frac{U_{1}}{2 \pi}\left[\frac{\xi q}{R(R+\eta)}+\tan ^{-1} \frac{\xi \eta}{q R}+I_{1} \sin \delta\right] \| \\
u_{y}=-\frac{U_{1}}{2 \pi}\left[\frac{\tilde{y} q}{R(R+\eta)}+\frac{q \cos \delta}{R+\eta}+I_{2} \sin \delta\right] \| \\
u_{z}=-\frac{U_{1}}{2 \pi}\left[\frac{\tilde{d} q}{R(R+\eta)}+\frac{q \sin \delta}{R+\eta}+I_{4} \sin \delta\right] \|
\end{array}\right.
$$

For dip-slip

$$
\left\{\begin{array}{l}
u_{x}=-\frac{U_{2}}{2 \pi}\left[\frac{q}{R}-I_{3} \sin \delta \cos \delta\right] \| \\
u_{y}=-\frac{U_{2}}{2 \pi}\left[\frac{\tilde{y} q}{R(R+\xi)}+\cos \delta \tan ^{-1} \frac{\xi \eta}{q R}-I_{1} \sin \delta \cos \delta\right] \| \\
u_{z}=-\frac{U_{2}}{2 \pi}\left[\frac{\tilde{d q}}{R(R+\xi)}+\sin \delta \tan ^{-1} \frac{\xi \eta}{q R}-I_{5} \sin \delta \cos \delta\right] \|
\end{array}\right.
$$


where

$$
\left\{\begin{array}{l}
I_{1}=\frac{\mu}{\lambda+\mu}\left[\frac{-1}{\cos \delta} \frac{\xi}{R+d}\right]-\frac{\sin \delta}{\cos \delta} I_{5} \\
I_{2}=\frac{\mu}{\lambda+\mu}[-\ln (R+\eta)]-I_{3} \\
I_{3}=\frac{\mu}{\lambda+\mu}\left[\frac{1}{\cos \delta} \frac{\tilde{y}}{R+d}-\ln (R+\eta)\right]+\frac{\sin \delta}{\cos \delta} I_{4} \\
I_{4}=\frac{\mu}{\lambda+\mu} \frac{1}{\cos \delta}[\ln (R+\tilde{d})-\sin \delta \ln (R+\eta)] \\
I_{5}=\frac{\mu}{\lambda+\mu} \frac{2}{\cos \delta} \tan ^{-1} \frac{\eta(X+q \cos \delta)+X(R+X) \sin \delta}{\xi(R+X) \cos \delta}
\end{array}\right.
$$

## STATIC DEFORMATION OF OCEAN BOTTOM

## EXAMPLE: VALPARAISO, CHILE

17 AUGUST 1906
$M_{0}=2.8 \times 10^{28}$ dyn-cm

$\phi_{f}=3^{\circ} ; \delta=15^{\circ} ; \lambda=117^{\circ}$
$L_{F}=200 \mathrm{~km} ; \quad W=75 \mathrm{~km} ;$

$$
\Delta u=5.3 \mathrm{~m}
$$

1906 CHILEAN EVENT

[Okal, 2005]

- Use this static deformation field (limited to its oceanic portion) as the initial condition $\left(t=0_{+}\right)$of the hydrodynamic calculation.
$\rightarrow$ Justification: The seismic source is generally MUCH FASTER than any tsunami process, hence it can be taken as instantaneous.

> (even in the case of SLOW, so-called "Tsunami" earhtquakes)

## PRODUCTS OF SIMULATION

CHILE $1906+10 \mathrm{hr}$.

1. Snapshots of Sea Height at Given Times

CHILE $1906+1 \mathrm{hr} .45 \mathrm{mn}$


## PRODUCTS OF SIMULATION

## 2. Map of Maximum Amplitude of Tsunami Wave




## HOW ROBUST IS THIS PROCEDURE?

It is worth exploring the robustness of our results in the far field, with respect to detailed parameters of our sources, a fortiori unknown in the context of many simulations.

We study simulated amplitudes of the 2004 Sumatra-Andaman tsunami in the far field under fluctuations of source parameters, while keeping the seismic moment of the source constant.

## We conclude that our results are indeed robust.

The primary parameters controlling the far field tsunami amplitudes are the size (moment) of the parent earthquake and the depth of the water column in the epicentral area.

1. MOVE SOURCE

LATERALLY

Move $1^{\circ}$ West


Move $1^{\circ}$ South


Move $1^{\circ}$ North


NO MAJOR EFFECT !!

## 2. CHANGE SOURCE PARAMETERS

SUMATRA 2004 Original


Heterogeneous Slip


Depth
SUMATRA 2004; D = 20 km

$\begin{array}{llllllllll}0.05 & 0.10 & 0.20 & 0.30 & 0.40 & 0.50 & 1.00 & 2.00 & 3.00 & 5.00\end{array}$

Fault Dip
SUMATRA 2004 Dip = 12 deg.


Strain Released
SUMATRA 2004 Large Strain


AMPLITUDE (m)

## By CONTRAST, WATER DEPTH at the SOURCE PLAYS a CRUCIAL ROLE

 NOTE: This explains the much smaller tsunami during the 2005 Nias earthquake.
## UNPERTURBED EPICENTRAL BATHYMETRY



EPICENTRAL BATHYMETRY DIVIDED BY 4.0



## NORMAL MODE FORMALISM: A different approach

[Ward, 1980]

- At very long periods (typically 15 to 54 minutes), the Earth, because of its finite size, can ring like a bell.
- Such FREE OSCILLATIONS are equivalent to the superposition of two progressive waves travelling in opposite directions along the surface of the Earth.

$$
\mathbf{T}=\mathbf{5 4} \text { minutes }
$$


"FOOTBALL Mode"

[After Lay and
Wallace, 1995]

T $=21.5$ minutes

"BREATHING
Mode"


Ward [1980] has shown that Tsunamis come naturally as a special branch of the normal modes of the Earth, provided it is bounded by an ocean, and gravity is included in the formulation of its vibrations.

In the normal mode formalism, the solution of the vertical displacement (both in the water and solid Earth) is sought as

$$
u_{z}(\mathbf{x} ; t)=u_{z}(r, \theta, \phi ; t)=y_{1}(r) \cdot Y_{l}^{m}(\theta, \phi) \exp (i \omega t)=y_{1}(r) \cdot P_{l}^{m}(\theta, \phi) \cdot e^{i m \phi} \cdot \exp (i \omega t)
$$

where $Y_{l}^{m}$ is a spherical harmonic of order $l$ and degree $m ; P_{l}^{m}$ is the Legendre polynomial of order $l$ and degree $m$; and $\{r, \theta, \phi\}$ is a system of spherical polar coordinates.

This allows for the separation of the variables $\{r, \theta, \phi\}$.
The problem is complemented by similar expressions for the overpressure $p=-y_{2}$ in the tsunami wave, the horizontal displacement $u_{x}=l \cdot y_{3}$, and the change in the gravity potential $y_{5}$.

Under the linear approximation, the equations of hydrodynamics transform into a system of linear differential equations of the first order.

For any given $l$, i.e., wavenumber $k=(l+1 / 2)$ ( $a$ radius of the Earth), the system has non trivial solutions for only one value of $\omega$. The relationship between $l$ and $\omega$ is the Disppersion Relation of the Tsunami.

## SPHEROIDAL MODE HAS 6-COMPONENT EIGENFUNCTION SATISFYING:

$y_{1}$ : Vertical displacement
$y_{3}$ : Horizontal displacement
$y_{2}$ : Normal stress
$y_{4}$ : Tangential stress
$y_{5}$ : Gravity potential
$y_{6}$ : Auxiliary gravity

## EIGENFUNCTIONS of SPHEROIDAL MODES

$$
\begin{aligned}
& \text { Rayleigh Mode } \\
& l=200 ; T=52 \mathrm{~s}
\end{aligned}
$$



Tsunami Mode
$l=200 ; T=908 s$
$y_{3}$ Horizontal Displacement
$y_{2}$ Pressure


TSUNAMI EIGENFUNCTION is CONTINUED (SMALL) into SOLID EARTH

## EXCITATION OF TSUNAMI in NORMAL MODE FORMALISM

- Gilbert [1970] has shown that the response of the Earth to a point source consisting of a single force $\mathbf{f}$ can be expressed as a summation over all of its normal modes

$$
\mathbf{u}(r, t)=\sum_{N} \mathbf{s}_{n}(\mathbf{r})\left(\mathbf{s}_{n}^{*}\left(\mathbf{r}_{\mathbf{s}}\right) \cdot \mathbf{f}\left(\mathbf{r}_{\mathbf{s}}\right)\right) \cdot \frac{1-\cos \omega_{n} t \exp \left(-\omega_{n} t / 2 Q_{n}\right)}{\omega_{n}^{2}},
$$

the EXCITATION of each mode being proportional to the scalar product of the force $\mathbf{f}$ by the eigen-displacement $\mathbf{s}$ at location $\mathbf{r}_{\mathbf{s}}$.

- Now, an EARTHQUAKE is represented by a system of forces called a double - couple:


The response of the Earth to an earthquake is thus

$$
\mathbf{u}(r, t)=\sum_{N} \mathbf{s}_{n}(\mathbf{r})\left(\varepsilon_{n}^{*}\left(\mathbf{r}_{\mathbf{s}}\right): \boldsymbol{M}\left(\mathbf{r}_{\mathbf{s}}\right)\right) \cdot \frac{1-\cos \omega_{n} t \exp \left(-\omega_{n} t / 2 Q_{n}\right)}{\omega_{n}^{2}}
$$

where the EXCITATION is the scalar product of the earthquake's MOMENT $M$ with the local eigenstrain $\varepsilon$ at the source $\mathbf{r}_{\mathrm{s}}$.

This formula is directly applicable to the case of a tsunami represented by normal modes of the Earth.

## ADVANTAGES of NORMAL MODE FORMALISM

- Handles any Ocean-Solid Earth Coupling Including Sedimentary Layers
- Works well at Higher Frequencies

No need to assume Shallow-Water Approximation

## DRAWBACKS of NORMAL MODE FORMALISM

- Must assume Laterally Homogeneous Structure
- Linear Theory -- Does not allow for Large Amplitudes

(2)

work of pressure
forces upon deformation: $W=\rho_{w} S g H \delta h$
(Also, move writer block a height $H$ )

NOTE: Energy scales as $L^{4}$, ie., as $M_{0}^{4 / 3}$.

## ENERGY of a TSUNAMI -- STATIC THEORY [Kajiura, 1981]

$$
E=\frac{1}{2} \frac{\rho_{w} g}{\mu^{2}} \alpha^{2 / 3} \cdot F(\delta, \lambda, h, R) \cdot M_{0}^{4 / 3}=\frac{1}{2^{4 / 3}} \frac{\rho_{w} g}{\mu^{4 / 3}} \varepsilon_{\max }^{2 / 3} \cdot F \cdot M_{0}^{4 / 3}
$$

* $\quad \alpha=$ invariant ratio of $M_{0}$ to $S^{3 / 2}$
* $\quad F$ : dimensionless factor expressing geometry of faulting, and aspect ratio $R$ of fault rupture area.

NOTE: Energy of Tsunami grows faster than Seismic Moment
Energy released by rupture, proportional to $\mathbf{M}_{\mathbf{0}}: \varepsilon$ grows like moment.

Hence, Fraction of Earthquake Energy transferred to Tsunami Grows with Earthquake Size
Fortunately, it remains VERY SMALL
(max. 1.3\% for Chile, 1960)

## TSUNAMI ENERGY COMPUTED from NORMAL MODE THEORY

[Okal, 2003]

- Compute Kinetic Energy of water in Normal Mode Formalism

Note that most energy is carried by HORIZONTAL FLOW

Weigh by excitation function for each mode for given seismic moment $M_{0}$. (averaged over focal geometry)

- Sum over individual modes (equivalent to integrating over frequency)

Account for source spectrum (according to seismic scaling laws)
Account for Finite extent of source depth.

$$
E=0.219 \frac{\rho_{w} g}{\mu^{4 / 3}} \cdot \varepsilon_{\max }^{2 / 3} \cdot M_{0}^{4 / 3}
$$

Essentially Equivalent to Kajiura's.

$$
E \text { grows as } M_{0}^{4 / 3}
$$

Sumatra 2004: $E \approx 7.5 \times 10^{23} \mathrm{erg}$ (100 times Hiroshima)

## WHAT ABOUT THE ATMOSPHERE?

If the tsunami eigenfunction is prolonged into the Solid Earth which is not totally rigid,

- It should be possible to prolong it into the atmosphere, which is not a perfect vacuum.
(The sea surface is not a totally "free" boundary)
- This idea, hinted at by Yuen et al. [1970], was proposed by Peltier [1976].


## $M_{\text {TSU }}$

- Use high seas tsunami waveforms recorded by DART system
- Consider tsunami as free oscillation branch of Earth's normal modes [Ward, 1980]
- Recall Magnitude $M_{m}$ for seismic mantle waves; Define


$$
M_{T S U}=\log _{10} X(\omega)+C_{D}+C_{S}+C_{0}
$$

Then, $\log _{10} M_{0}=M_{\text {TSU }}+20$

- IT WORKS !!
[Okal and Titov, 2006]



## RECALL MANTLE MAGNITUDE

 [Okal and Talandier, 1989]$$
M_{m}=X(\omega)+C_{D}+C_{S}+C_{0}
$$

- Applied to mantle Rayleigh waves; typically, $T=50$ to 300 seconds.
- $X(\omega)$ is spectral amplitude in $\mu \mathrm{m}$ *s
- $\quad C_{D}$ is distance correction
- $\quad C_{S}$ is source (frequency) correction
- $C_{0}=-0.90$ is locking constant (predicted theoretically)

THEN, $\quad M_{m}$ is directly related to seismic moment $M_{0}$ :

$$
M_{m}=\log _{10} M_{0}-20
$$

$M_{m}$ combines simple "quick-and-dirty" concept of one-station magnitude with modern analytical approach (measuring a bona fide physical quantity, the seismic moment, using physical units). It does not saturate.

Valid even for 1960 Chilean earthquake.
A tsunami wave on the high seas is a branch of normal
modes of the Earth [Ward, 1980].
$\rightarrow$ QUESTION: Can we extend the concept of $M_{m}$ to a tsunami wave measured on the high seas -- and call it $M_{T S U}$ ?

## DEVELOPING A FORMULA FOR $M_{T S U}$

The basic formula for the spectral amplitude of a spheroidal wave by a dislocation remains applicable:
$X(\omega)=M_{0} \cdot a \sqrt{\frac{\pi}{2}} \cdot \frac{1}{\sqrt{\sin \Delta}} e^{-\frac{\omega a \Delta}{2 U Q}} \cdot\left[\frac{1}{U}\left|s_{R} l^{-1 / 2} K_{0}-p_{R} l^{3 / 2} K_{2}-i q_{R} l^{1 / 2} K_{1}\right|\right]$

$$
M_{m}=\log _{10} M_{0}-20=\log _{10} X(\omega)+C_{D}+C_{S}+C_{0}
$$

Need only adjust the corrections $C_{D}$ and $C_{S}$ and the constant $C_{0}$.

## THE DISTANCE CORRECTION $C_{D}$

$$
C_{D}=\frac{1}{2} \log _{10} \sin \Delta
$$

## THE SOURCE (FREQUENCY) CORRECTION $C_{S}$

$$
\begin{gathered}
C_{S}=-\log _{10}\left[\frac{\langle | s_{R}-p_{R} \mid>\omega^{1 / 2} g^{-3 / 4}}{8 \pi \mu a^{3 / 2}} \cdot H^{-3 / 4}\right] \\
C_{S}=0.087 \theta^{3}-0.069 \theta^{2}+0.508 \theta+2.299 \\
\left(\theta=\log _{10} T-3.122\right)
\end{gathered}
$$

(The latter formula uses Okal's [2003] asymptotic expressions of the tsunami eigenfunction to compute the various excitation coefficients for a shallow source in the limit $\omega \rightarrow 0$ ).

## DEVELOPING A FORMULA FOR $M_{T S U}$

The basic formula for the spectral amplitude of a spheroidal wave by a dislocation remains applicable:
$X(\omega)=M_{0} \cdot a \sqrt{\frac{\pi}{2}} \cdot \frac{1}{\sqrt{\sin \Delta}} e^{-\frac{\omega a \Delta}{2 U Q}} \cdot\left[\frac{1}{U}\left|s_{R} l^{-1 / 2} K_{0}-p_{R} l^{3 / 2} K_{2}-i q_{R} l^{1 / 2} K_{1}\right|\right]$

$$
M_{m}=\log _{10} M_{0}-20=\log _{10} X(\omega)+C_{D}+C_{S}+C_{0}
$$

Need only adjust the corrections $C_{D}$ and $C_{S}$ and the constant $C_{0}$.

## THE LOCKING CONSTANT $C_{0}$

If $X(\omega)$ is the spectrum of the wave height at the surface in cm*s, then

$$
C_{0}=3.10
$$

If one uses the bottom pressure $p(t)$ recorded in $\mathbf{d y n} / \mathbf{c m}^{2}$ on the ocean bottom, then use $P(\omega)$ rather than $X(\omega)$; $\left[P(\omega)=\rho_{w} g X(\omega)\right]$, and

$$
C_{0}=0.11
$$

If $p(t)$ is recorded in pounds[-force] per square inch, then

$$
C_{0}=4.95
$$

## Case Study: KURIL ISLANDS, 04-OCT-1994



WC62 KURILES 1994



## Case Study: KURIL ISLANDS, 04-OCT-1994



Published: 8.48; Mean $M_{T S U}=8.08 \pm 0.14$



Works despite UNFAVORABLE GEOMETRY requiring NON-GEOMETRICAL propagation !!

## SUCCESSFUL OPERATIONAL USE 17 NOV 2003

This is a smaller earthquake which was not recorded at the Alaskan and West Coast DART gauges.

However, a new station, D-171, is only 900 km from the epicenter, and clearly recorded the tsunami, although at a very coarse sampling (1 minute).

Despite this limitation, the event can be successfully processed.

D171 ALEUTIAN 17-NOV-2003


Frequency (Millihertz)

## $\rightarrow$ This estimate was used in real-time to call off an alert for Hawaii.

## APPLICATION of $M_{T S U}$ to JASON SATELLITE TRACE

DETECTION by SATELLITE ALTIMETRY gives first definitive measurement of MAJOR tsunami on HIGH SEAS
(previous detection by Okal et al. [1999] during 1992 Nicaragua tsunami -- 8 cm -- at the limit of noise).


TRACE of ALTIMETRY SATELLITE OVER INDIAN OCEAN


Satellite at the right place at the right time!

## measures 70 cm across Bay of Bengal



QUESTION: Can we quantify the JASON trace, i.e., recover from it the source of the tsunami?

- PROBLEM: JASON is neither a time series nor a space series.
- SOLUTION : Rebuild an approximate times series from the JASON trace, then process through $M_{T S U}$.



## Original Jason Trace



Equivalent Time Series

$$
\begin{aligned}
& \text { ( } 50 \text { (c) }
\end{aligned}
$$

$$
\begin{aligned}
& \text { CONCLUSION: IT WORKS !! }
\end{aligned}
$$

## $M_{T S U}$ CONCLUSION

- The algorithm succesfully retrieves the seismic moment of the parent earthquake.

- The examples tested suggest that the precision is sufficient to avoid false alarms and failures to warn.

