LECTURE 3

EARTHQUAKES: DETECTION,

LOCATION & FOCAL GEOMETRY
EARTHQUAKE LOCATION

Retrieved *(Inverted)* from **arrival times** of BODY (principally $P$) waves

The problem consists of determining

\[
\begin{align*}
\text{Epicenter} & \{ \begin{align*}
* & \text{Latitude} \\
* & \text{Longitude} \\
[ & \text{Depth} \\
* & \text{Origin Time}
\end{align*} \} & \text{Hypocenter}
\end{align*}
\]
$P$ times are usually easiest to pick

**EXAMPLE:** JAPAN SEA Event, 14 NOV 2005

Station AAK (Ala Archa, Kyrgyzstan); $\Delta = 52.4^\circ$

Gather such data for many stations

→ Obtain dataset of *observed* arrivals \{ $o_i$ \}.  

\[
\begin{array}{llllll}
\text{AAKZ} & 05 & 318 & 21 & 45 & 40.8270 \\
\end{array}
\]
IN THE NEAR FIELD

- In the presence of a dense array, the
  
  *Easiest, Simplest, Crudest Algorithm*
  
  consists of identifying

  \[ \text{Epicenter} \approx \text{Station with Earliest Arrival} \]
IN THE FAR FIELD

• Position of problem:

Retrieve

Latitude $\lambda$
Longitude $\phi$
Depth $h$
Origin time $t_0$

from dataset $\{ o_i \}$.

• $P$–wave arrival times can be computed as functions of source and station parameters (Latitude and Longitude $\Lambda_i$, $\Phi_i$) using a model of Earth structure.

$$c_i = f ( \lambda, \phi, h, t_0; \Lambda_i, \Phi_i )$$

• DIFFICULTY:

Function $f$ is NON–LINEAR.
LINEARIZING the PROBLEM

• Assume *Trial Solution*

\[ \lambda^0, \phi^0, h^0, t^0_0 \]

and compute a set of *predicted arrival times* \{\(c_i\)\} for that solution, based on a chosen Earth model (*Jeffreys-Bullen*, *PREM*, *iaspei91*, etc.).

→ If all the data were perfect (no noise), as well as the model, and we had guessed the right solution, then for all \(i\), we should have \(c_i = o_i\).

• Define *RESIDUALS*

\[ \delta t_i = o_i - c_i = (o - c)_i \]

Hopefully, the \(\delta t_i\) are small compared with the propagation times \(c_i - t^0_0\).
LINEARIZING the PROBLEM (2)

- Then try improving the solution from \{\lambda^0, \phi^0, h^0, t_0^0\} to \{\lambda^1, \phi^1, h^1, t_0^1\}

\[
\begin{pmatrix}
\lambda^1 \\
\phi^1 \\
h^1 \\
t_0^1
\end{pmatrix} =
\begin{pmatrix}
\lambda^0 \\
\phi^0 \\
h^0 \\
t_0^0
\end{pmatrix} +
\begin{pmatrix}
\delta \lambda \\
\delta \phi \\
\delta h \\
\delta t_0
\end{pmatrix}
\]

Again, we expect the terms \delta \cdots to be small, so that the change in each \(c_i\) is simply

\[
\delta c_i = \frac{\partial f}{\partial \lambda} \cdot \delta \lambda + \frac{\partial f}{\partial \phi} \cdot \delta \phi + \frac{\partial f}{\partial h} \cdot \delta h + \frac{\partial f}{\partial t_0} \cdot \delta t_0
\]

- If we know the function \(f\) ("direct problem"), we should be able to compute the partial derivatives such as \(\frac{\partial f}{\partial \lambda}\).
**LINEARIZING the PROBLEM (3)**

- Ideally, we would like, for each station, $\delta c_i$ to be exactly $\delta t_i = (o - c)_i$, so we seek to solve

\[
\begin{bmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
\begin{bmatrix}
\delta c_i \\
\delta c_i \\
\delta c_i \\
\delta c_i \\
\delta c_i \\
\end{bmatrix}
= A
\begin{bmatrix}
\delta \lambda \\
\delta \phi \\
\delta h \\
\delta t_0 \\
\delta t_i \\
\end{bmatrix}
= 
\begin{bmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
\]
LINEARIZING the PROBLEM (4)

The problem has been *LINEARIZED* but it is still *OVER-DETERMINED* as $A$ is a very tall matrix (4 columns (4 unknowns) and tens or hundreds of rows (data points)).

→ It can be solved by the *classical LEAST–SQUARES algorithm*:

$$
\begin{bmatrix}
\delta \lambda \\
\delta \phi \\
\delta h \\
\delta t_0
\end{bmatrix}
= (A^T A)^{-1} \cdot A^T
\begin{bmatrix}
\cdots \\
\cdots \\
\cdots \\
\cdots
\end{bmatrix}
\delta t_i$$

• Note that $(A^T A)$ is a $4 \times 4$ matrix, and $A^T \cdot \delta t$ is a 4–dimensional vector.
DETAILED LOOK at the MATRIX $A$

The elements of $A$ are the partial derivatives of the arrival times $c_i$ at station $i$ with respect to a change in a source parameter

- **An easy case**

$$\frac{\partial c_i}{\partial t_0} = 1 \quad \text{for all } i$$

Otherwise, for a spherical Earth, the travel-time $t_P$ is function of the angular distance $\Delta$ to the station, and of the source depth, $h$.

- **Source depth**

$$\frac{\partial c_i}{\partial h} = - \frac{\cos j_i}{V_P(h)}$$

$j$ is itself a function of $\Delta$ and $h$

NOTE that, at teleseismic distances, $j$ is always a small angle...
• Latitude and Longitude

If we change the latitude by $\delta \lambda$, we move the epicenter North by an amount $\delta \lambda \cdot l_{\text{deg}}$, where $l_{\text{deg}} = 111.195$ km is the length of one degree at the Earth’s surface.

Thus, we change the distance to the station $i$ by $\delta \Delta_i = -\delta \lambda \cdot \cos \alpha_i$, and

$$\frac{\partial c_i}{\partial \lambda} = -\cos \alpha_i \cdot \frac{\partial T_P}{\partial \Delta}$$

If we change the longitude by $\delta \phi$, we move Eastwards, but only by $\delta \phi \cdot \cos \lambda \cdot l_{\text{deg}}$, so that

$$\frac{\partial c_i}{\partial \phi} = -\sin \alpha_i \cdot \cos \lambda \cdot \frac{\partial T_P}{\partial \Delta}$$
IN SUMMARY

• We can compute all the partial derivatives

• We can compute the matrix $A$

• We can compute $(A^T A)$ and invert it

• We can find the "best" change in earthquake source parameters to minimize the new residuals

• We can iterate the process until the solution stabilizes
LIMITATIONS of THIS ALGORITHM

• The matrix can be inverted only if it is
  \[\text{NON} – \text{SINGULAR}\]

  [ in practice \text{NOT APPROACHING SINGULARITY} ]

• The matrix is singular if 2 rows (or columns) are identical.

• Recall

\[
\frac{\partial c_i}{\partial t_0} = 1 \quad \text{and} \quad \frac{\partial c_i}{\partial h} = -\frac{\cos j_i}{V^P(h)}
\]

If all the stations are at the same distance, then all \( j_i \) are the same and the two partials are proportional.

\text{SINGULARITY}!

• In practice, if all stations are \emph{far away}, then all \( j_i \) are small ( \(< 10^\circ\); rays all take off nearly vertically at the source), all \( \cos j_i \approx 1 \), and one has

\text{PERFECT TRADE-OFF BETWEEN O.T. and DEPTH}

In general, the inversion becomes unstable. The only way out is to

\text{CONSTRAIN the DEPTH}...
SIMILARLY

• Recall

\[ \frac{\partial c_i}{\partial \lambda} = - \cos \alpha_i \cdot \frac{\partial T_P}{\partial \Delta} \]

and

\[ \frac{\partial c_i}{\partial \phi} = - \sin \alpha_i \cdot \cos \lambda \cdot \frac{\partial T_P}{\partial \Delta} \]

If all stations are in [approximately] the same azimuth, the two columns of partials are proportional, and the matrix features [or approaches] singularity.

→ \text{STABLE LOCATIONS REQUIRE A GOOD AZIMUTHAL COVERAGE}

[This is usually not an issue for large events ]
INFLUENCE of TRIAL SOLUTION

- If dataset is global, any trial solution (even the antipodes of the true epicenter) will lead to a converging algorithm [Okal and Reymond, 2003].

- In practice, one can always use the station with earliest arrival as a trial epicenter.
ONE-STATION ALGORITHMS

Detection and Location

→ Enhance performance of single station/observatory

• Detection Algorithms

Generally based on the monitoring of energy in the ground (or velocity) motion at the station.

* Define a **short-term average** over a short sliding window

* Compare it with a **delayed long-term average**.

![Graph showing short-term (STA) and long-term (LTA) averages]

• When \( \frac{E_{\text{in STA}}}{E_{\text{in LTA}}} \) exceeds a given threshold,

**Trigger Detection**

* Earthquake spectrum is generally **white**, so do this in **several frequency bands**.

Coherence across spectrum is required to trigger detection

[or across several stations of a local network, when available to prevent triggering on human noise.]

→ Arrival times \( o_i \) can be defined by evolution of \( E_{\text{STA}}/E_{\text{LTA}} \).
SINGLE-STATION LONG-PERIOD LOCATION

• **IDEA ONE**

"$S - P$" interval between $P$ and $S$ waves can give DISTANCE

\[
S - P = 577 \text{ s.}
\]

\[
\Delta = 74.9 \text{ deg.}
\]
SINGLE-STATION LONG-PERIOD LOCATION

- **IDEA TWO**

Polarization of $P$ wave can give **AZIMUTH of ARRIVAL**, $\beta$

$\beta = 33$ deg.
SINGLE-STATION LONG-PERIOD LOCATION

- Combine \textit{DISTANCE} and \textit{BACK AZIMUTH} to obtain

\textbf{Estimate of Epicenter}

\textbf{14 NOV 2005}
EXAMPLE of SINGLE-STATION LONG-PERIOD LOCATION TREMORS — Reymond et al. [1991]

Earthquake located about 300 km from true epicenter


**EARTHQUAKE SOURCE GEOMETRY**

*From Single Force to Double-Couple*

The physical representation of an earthquake source is a system of forces known as a *Double-Couple*, the direction of the forces in each couple being the direction of slip on the fault and the direction of the normal to the fault plane.

Mathematically, the system of forces is described by a *Second-Order Symmetric Deviatoric Tensor* (3 angles and a scalar).

[Stein and Wysession, 2002]
The focal geometry of earthquakes can vary depending on the orientation of the double-couple representing the source. Here are some basic examples:

[Stein and Wysession, 2002]

**HOW CAN WE**

- Best describe this geometry?
- Determine it from seismological data?
- Represent it graphically in simple terms?
EARTHQUAKE SOURCE GEOMETRY

THREE ANGLES are necessary to describe the focal mechanism of an earthquake:

- The strike angle $\phi$ identifies the azimuth of the trace of the fault on the horizontal Earth surface;
- The dip angle $\delta$ indicates how steeply the fault penetrates the Earth;
- The slip angle $\lambda$ describes the relative motion of the two blocks on the fault plane determined by $\phi$ and $\delta$.

$\rightarrow$ The physical description of an earthquake source is thus more complex than a vector since it requires three angles as opposed to two.

[Stein and Wysession, 2002]
The strike angle $\phi$

(between $0^\circ$ and $360^\circ$)

defines the azimuth of the trace of the fault on the Earth’s surface

(the orientation of the knife)
The dip angle $\delta$

$(between \ 0^\circ \ and \ 90^\circ)$

defines the slope (dip) of the fault to be cut through the material

(the \emph{inclination of the blade} on the horizontal)

\begin{itemize}
  \item \textbf{Vertical dip} \quad (\delta = 90^\circ)
  \item \textbf{Shallow dip} \quad (\delta = 30^\circ)
\end{itemize}
The slip (or rake) angle $\lambda$

$(between \ 0^\circ \ and \ 360^\circ)$

defines the direction of motion of the blocks on the fault plane (cut) defined by $\phi$ and $\delta$.

Strike-slip ($\lambda = 180^\circ$)

No vertical motion

Dip-slip ($\lambda = 270^\circ$)

Motion along line of steepest descent
Varying the slip (or rake) angle $\lambda$ (ctd.)

**Thrust Faulting** ($\lambda = 90^\circ$)  
*(Typical of subduction zones)*

**Normal Faulting** ($\lambda = 270^\circ$)  
*(Typical of tensional environments)*
Varying the slip (or rake) angle $\lambda$ (ctd.)

**HYBRID MECHANISMS**

**Thrust and Strike-slip** ($\lambda = 120^\circ$)

**Normal Faulting and Strike-Slip** ($\lambda = 315^\circ$)
Double-Couple mechanisms give rise to $P$ waves which can have positive (first-motion "up") or negative (first-motion "down") initial motions.

The repartition of such motions on a small sphere surrounding the source involves four alternating quadrants in space.

We represent focal mechanisms by giving a stereographic view of a small focal [hemi]sphere with positive quadrants shaded and negative ones left open.

These are called "focal beachballs".

[Stein and Wysession, 2002]
EXAMPLES of EARTHQUAKE SOURCE GEOMETRIES

\[ \lambda = 90^\circ \quad \text{Pure dip-slip (thrust)} \]

\[ \lambda = 120^\circ \quad \text{Mostly dip-slip with some strike-slip} \]

\[ \lambda = 150^\circ \quad \text{Mostly strike-slip with some dip-slip} \]

\[ \lambda = 180^\circ \quad \text{Pure strike-slip (right lateral)} \]

\[ \lambda = 210^\circ \quad \text{Mostly strike-slip with some dip-slip} \]

\[ \lambda = 240^\circ \quad \text{Mostly dip-slip with some strike-slip} \]

\[ \lambda = 270^\circ \quad \text{Pure dip-slip (normal)} \]

[Stein and Wysession, 2002]
ALL FOCAL MECHANISMS ARE CREATED EQUAL...

They are just ONE SOLID ROTATION away from Each Other

Strike-Slip

Vertical Dip-Slip

Thrust

Normal

Hybrid
DETERMINATION of FOCAL MECHANISMS

- Historically

Examine first motion of $P$ waves and plot them on a beach ball.

[Stein and Wysession, 2002]
DETERMINATION of FOCAL MECHANISMS

• *Modern Technique*

Directly invert waveforms at many stations for the *components of the moment tensor* representing the double-couple.

"Centroid Moment Tensor Inversion"

→ This is possible because that seismic ground displacement is a linear combination of these components.

**Body Waves**

\[ u_n(x; t) = M_{pq} \ast G_{np,q} = \frac{\gamma_n \gamma_p \gamma_q}{4 \pi \rho \alpha^3 r} \frac{\partial M_{pq}}{\partial \alpha} \left( t - \frac{r}{\alpha} \right) \]

**P waves**

**S waves**

\[ P \text{ waves} \quad S \text{ waves} \]

Normal modes

\[ u(r, t) = \sum N \mathbf{s}_n(r) \left( \mathbf{e}_n^*(r_s) : \mathbf{M}(r_s) \right) \cdot \frac{1 - \cos \omega_n t \exp(-\omega_n t/2Q_n)}{\omega_n^2} \]

**NOTE LINEARITY of all Equations with respect to** \( M_{pq} \).
NEAR-REAL TIME CMT SOLUTIONS

Computed routinely by the NEIC (body waves) and the Global CMT (ex-Harvard) project (body and surface waves)

Example: **08 JUL 2007, ALEUTIAN ISLANDS**

<table>
<thead>
<tr>
<th>COMPANY</th>
<th>EVENT</th>
<th>MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEIC</td>
<td></td>
<td>6.6</td>
</tr>
<tr>
<td>Global CMT</td>
<td></td>
<td>6.7</td>
</tr>
</tbody>
</table>

**NEIC**

07/08/02 03:21:46.54
ANDREANOF ISLANDS, ALEUTIAN IS.
Epicenter: 51.358 -179.929
MW 6.6

USGS MOMENT TENSOR SOLUTION
Depth: 21
No. of sta: 54
Moment Tensor: Scale 10**19 Nm
Mrr= 0.73
Mtt= -0.59
Mpp= -0.14
Mrt= 0.53
Mrp= 0.41
Mtp= -0.40
Principal axes:
T Val= 0.99 Plg= 69 Azm= 316
N 0.09 4 57
P -1.07 20 148

Best Double Couple: M0= 1.0*10**19
NP1: Strike= 246 Dip= 25 Slip= 100
NP2: 55 65 85

\[ M_0 = 1.0 \times 10^{26} \text{ dyn*cm} \]

**Global CMT**

August 2, 2007, ANDREANOF ISLANDS, ALEUTIAN IS., MW=6.7

Goran Ekstrom

CENTROID-MOMENT-TENSOR SOLUTION
GCMT EVENT: C200708020321A
DATA: II IU CU IC GE
L.P.BODY WAVES: 64S, 165C, T= 50
MANTLE WAVES: 62S, 138C, T= 125
SURFACE WAVES: 64S, 172C, T= 50
TIMESTAMP: Q-200708021040412

CENTROID LOCATION:
ORIGIN TIME: 03:21:51.2 0.1
LAT= 51.11N 0.00; LON= 179.66W 0.01
DEP: 32.2 0.2; TRIANG HDUR: 5.6
MOMENT TENSOR: SCALE 10**26 D-CM
RR= 1.010 0.006; TT= -1.050 0.005
PP= 0.031 0.005; RT= 0.740 0.010
RP= 0.716 0.010; TP= -0.403 0.004

PRINCIPAL AXES:
1. (T) VAL= 1.484; PLG= 64; AZM= 297
2. (N) 0.045; 15; 60
3. (P) -1.538; 21; 156

BEST DBLE.COUPLE: M0= 1.51*10**26
NP1: STRIKE= 271; DIP= 27; SLIP= 123
NP2: STRIKE= 54; DIP= 67; SLIP= 74

\[ M_0 = 1.5 \times 10^{26} \text{ dyn*cm} \]

Note difference in moment

The two focal solutions are separated by a solid rotation of **11°**.