Theoretical Seismology 2: Wave Propagation

Based on a lecture by James Mori of the Earthquake Hazards Division, Disaster Prevention Research Institute, Kyoto University

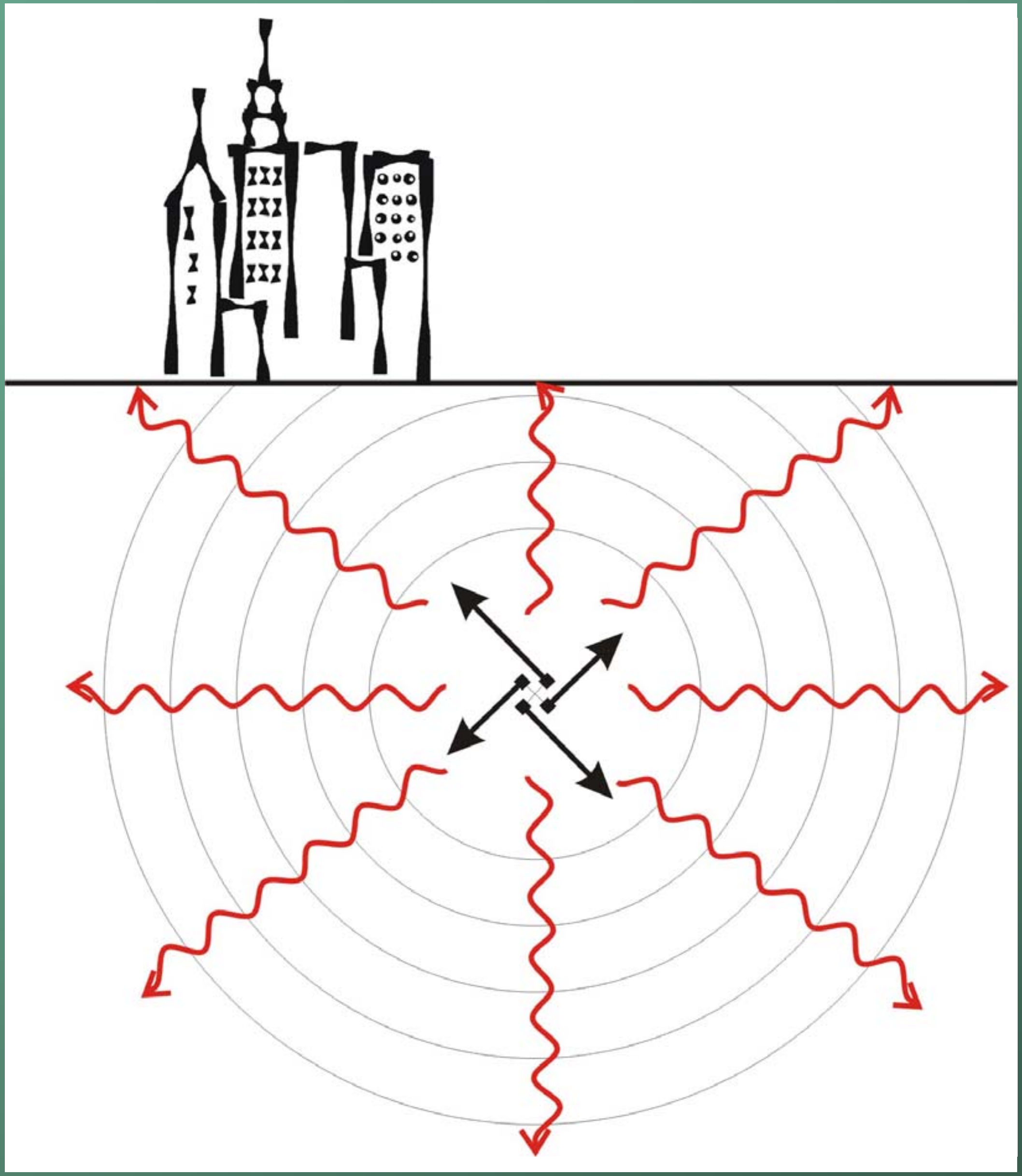
U.S. Department of the Interior
U.S. Geological Survey

Contents

- Rays
  Snell's Law
  Structure of the Earth

- Seismic Waves
  Near-Field Terms (Static Displacements)
  Far-Field Terms (P, S, Surface waves)

- Normal modes
  Free oscillations of the Earth



USGS

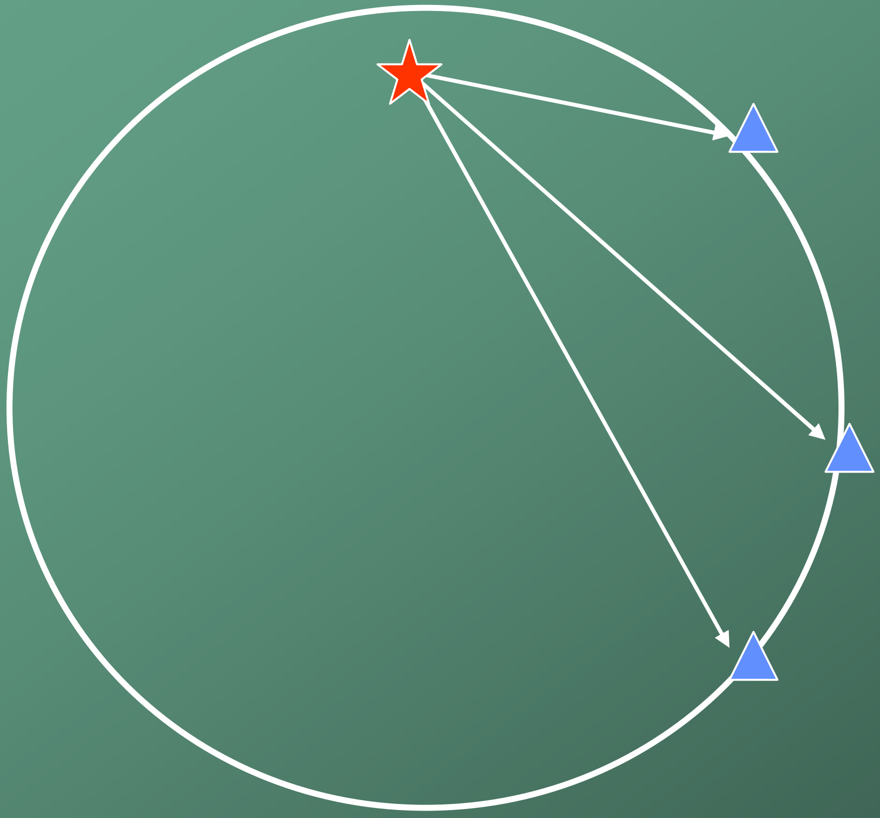
Faulting

↓

Seismic waves

USGS science for a changing world

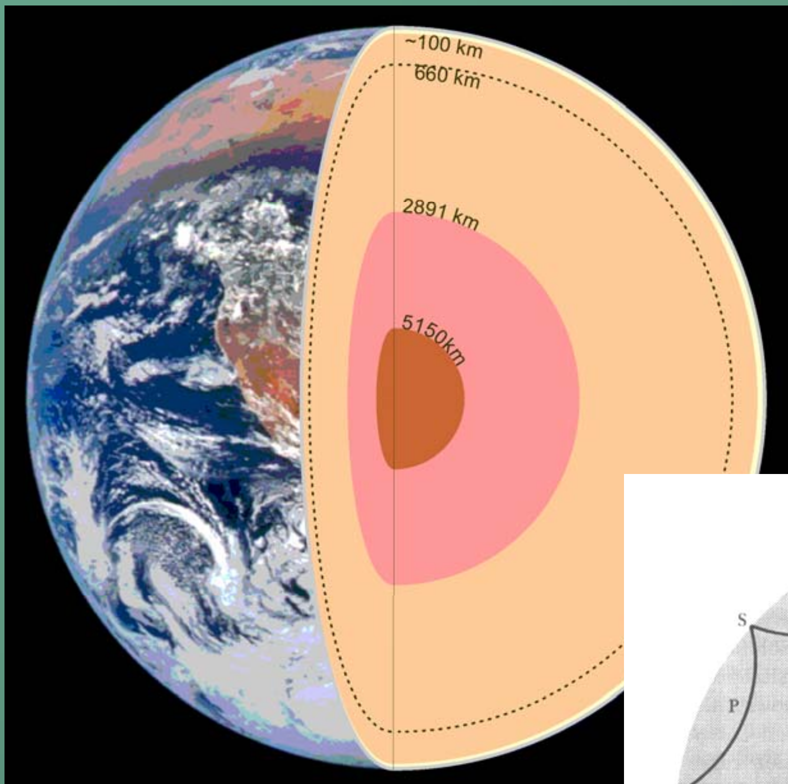
Homogeneous Earth



If the Earth had constant velocity the wave paths would be very simple.

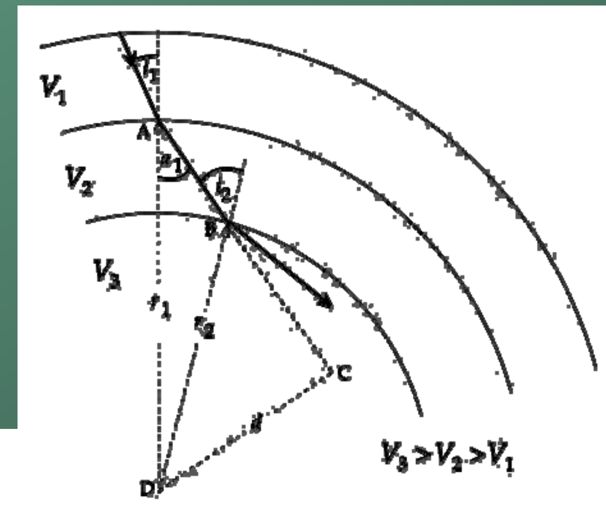
USGS
science for a changing world

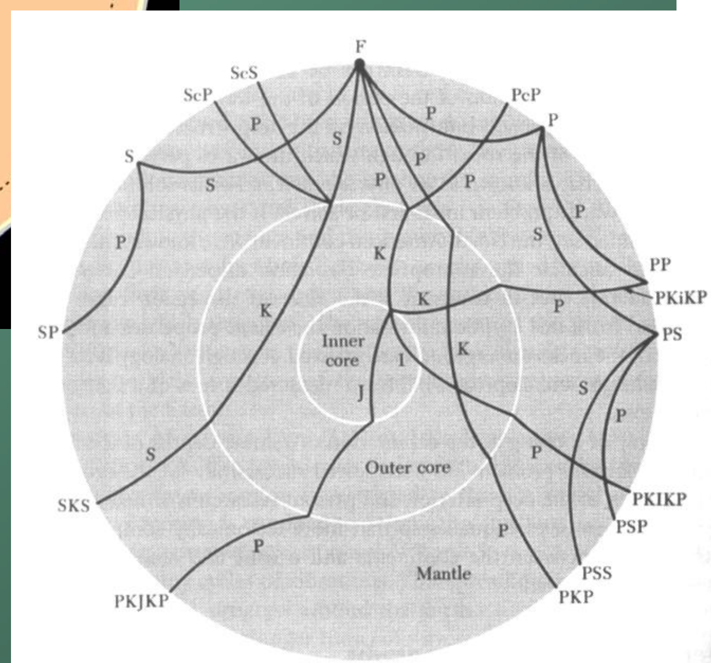
Structure in the Earth results in complicated paths



USGS

Lowrie, 1997, fig 3.69



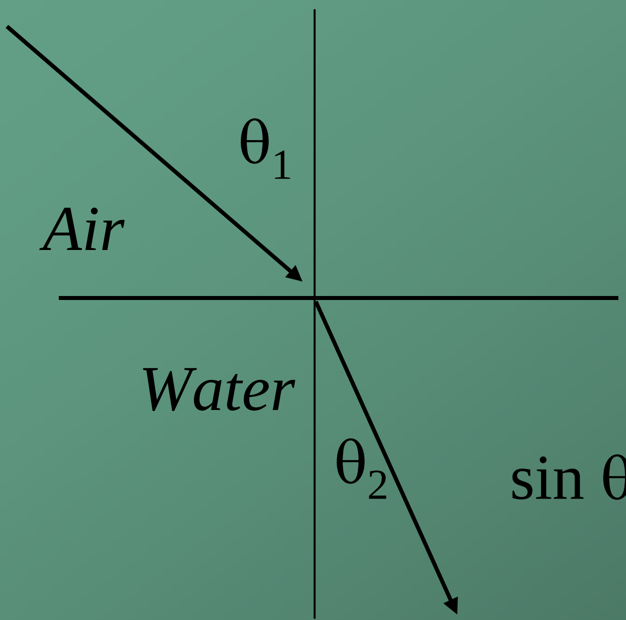


Bolt, 2004, fig 6.3

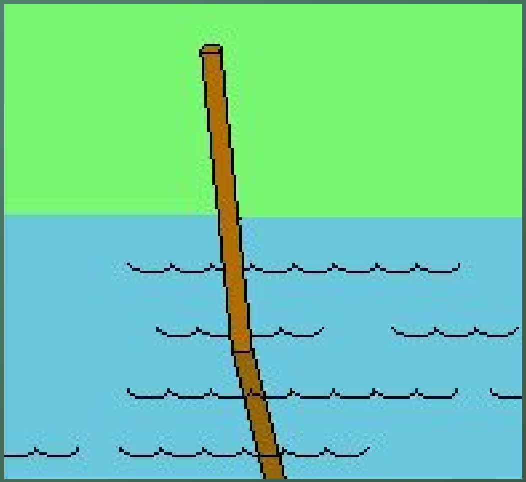
USGS
science for a changing world

Rays

Snell's Law
Fermat's Principle

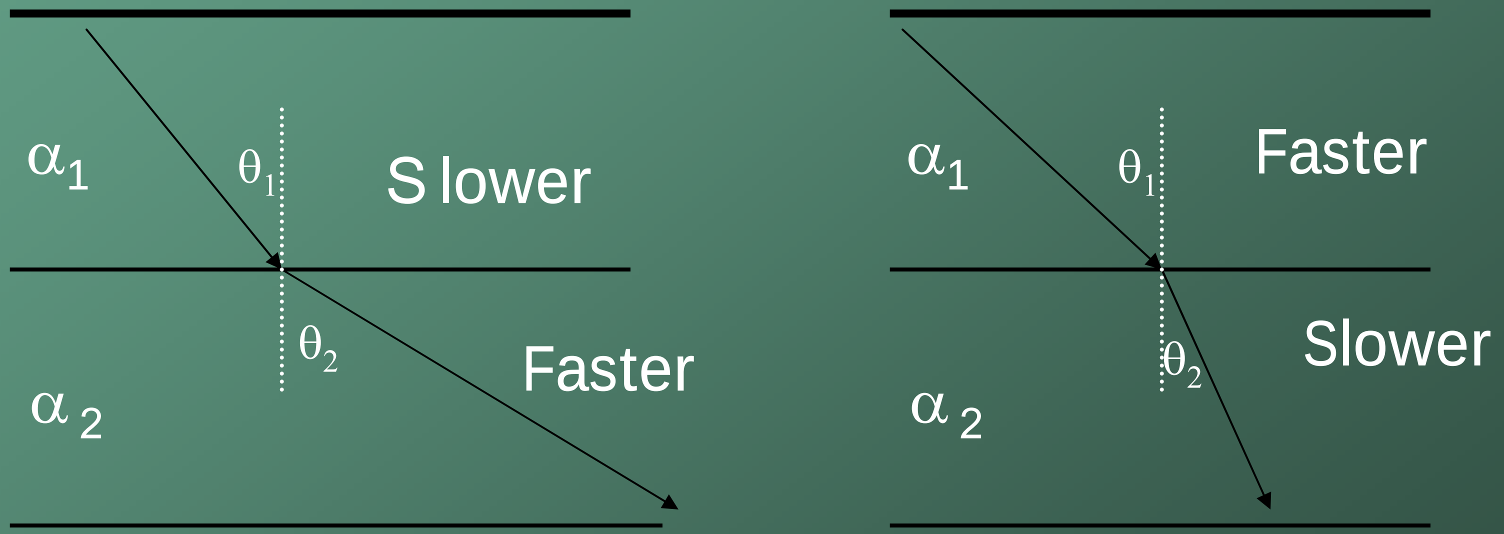


$$\sin \theta_1 / \sin \theta_2 = n_{21}$$



Ray Paths in a Layered Medium

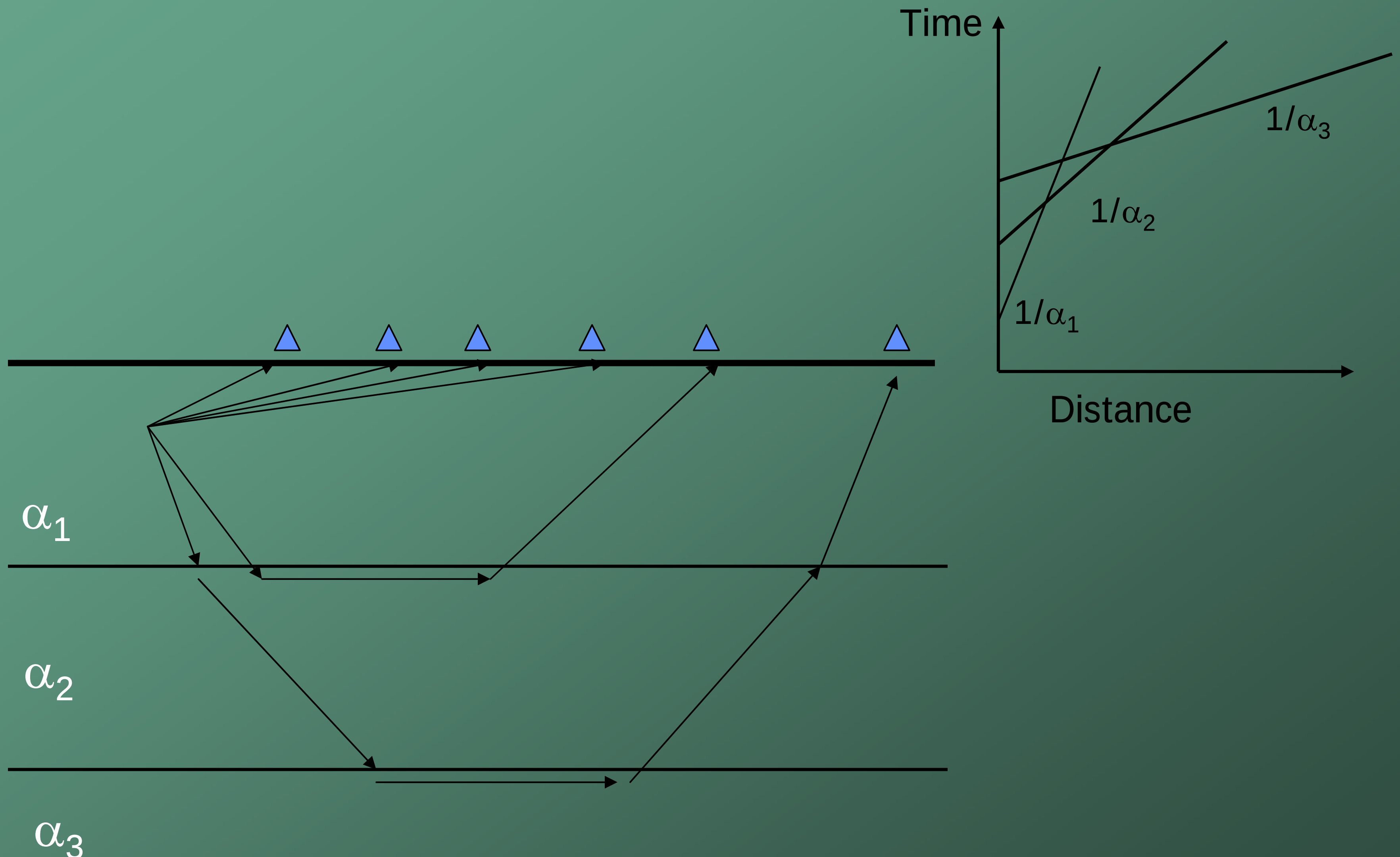
α = velocity of seismic energy in the layer



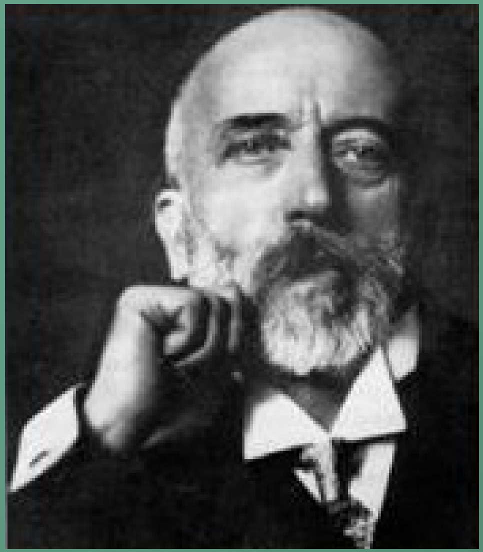
$\alpha_1 < \alpha_2$ $\alpha_1 > \alpha_2$

USGS
science for a changing world

Ray Paths in a Layered Medium



The Moho



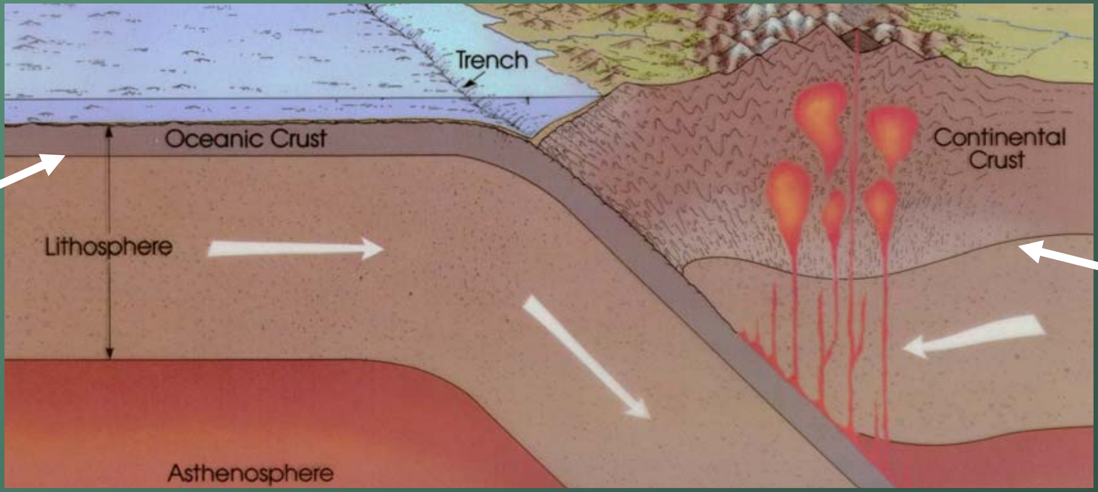
Andrija Mohorovicic (1857-1936)

Found seismic discontinuity at 30 km depth in the Kupa Valley (Croatia).

Mohorovicic discontinuity or ‘Moho’

Boundary between crust and mantle

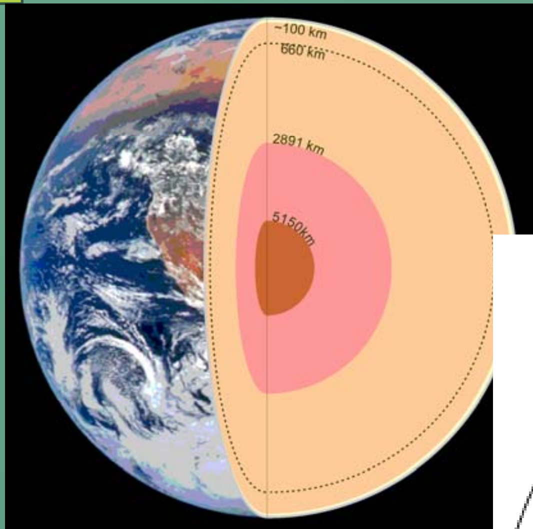
The Moho

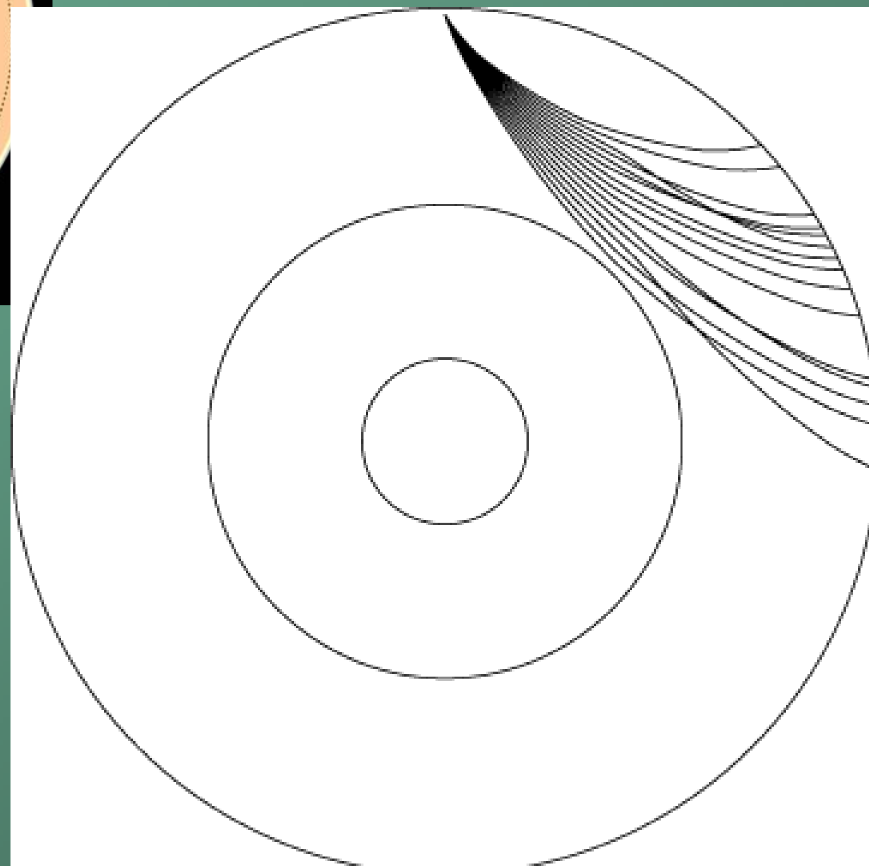


The Moho

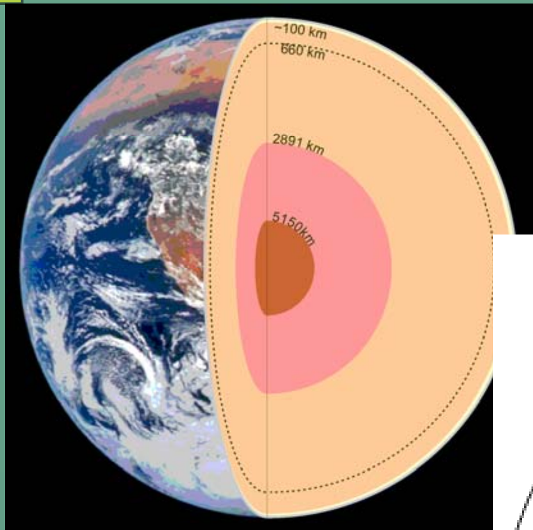
Copywrite Tasa Graphic Arts

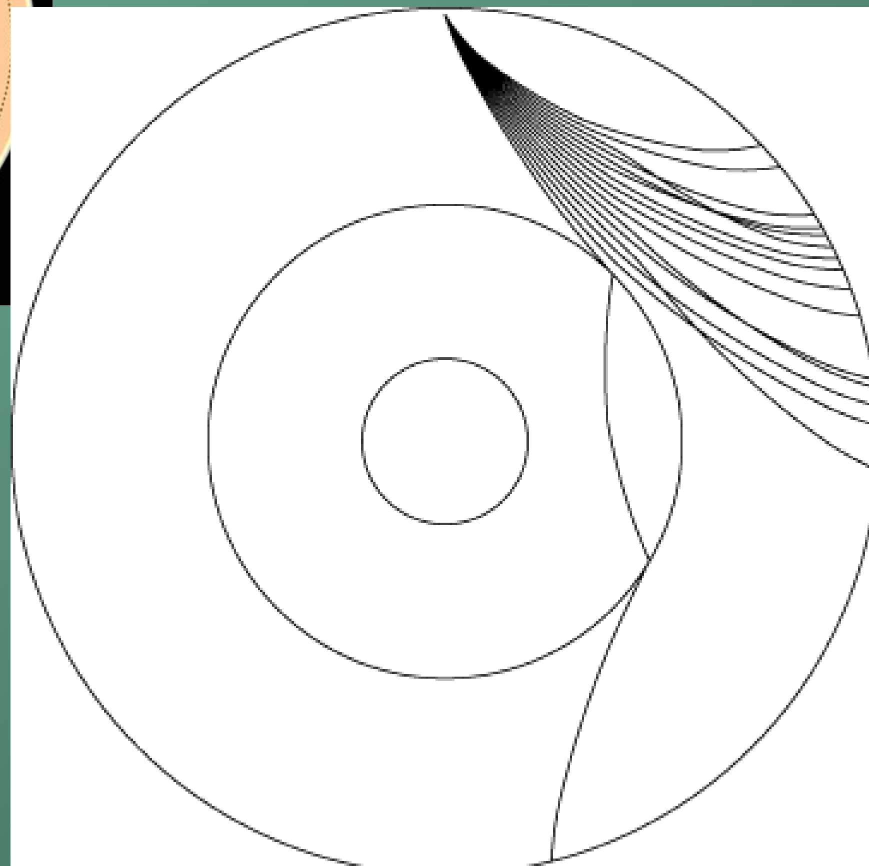
USGS science for a changing world



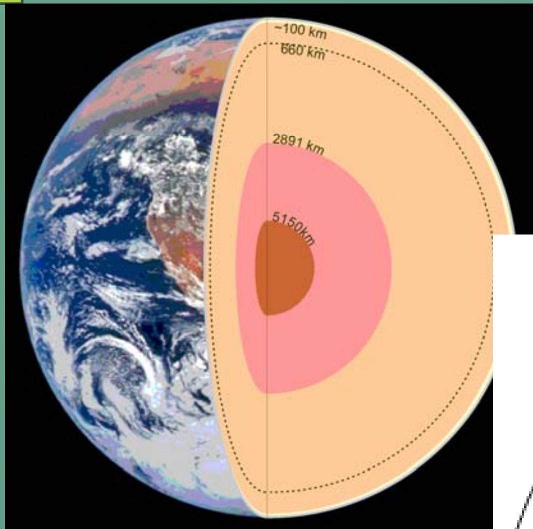


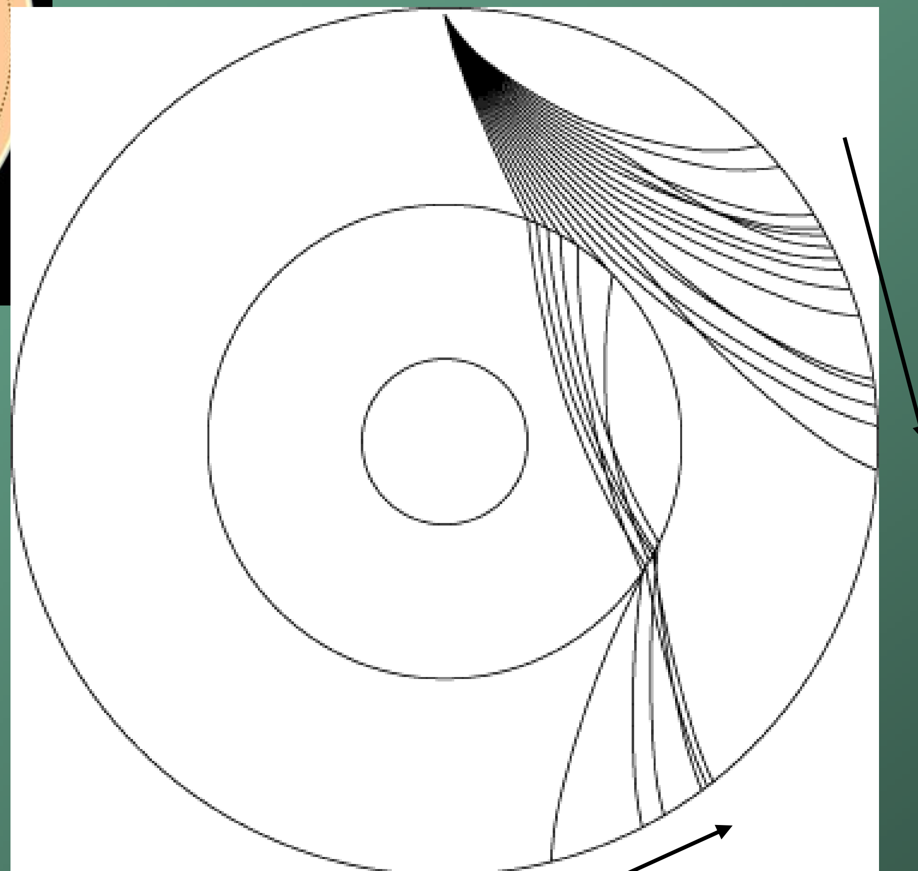
USGS
science for a changing world

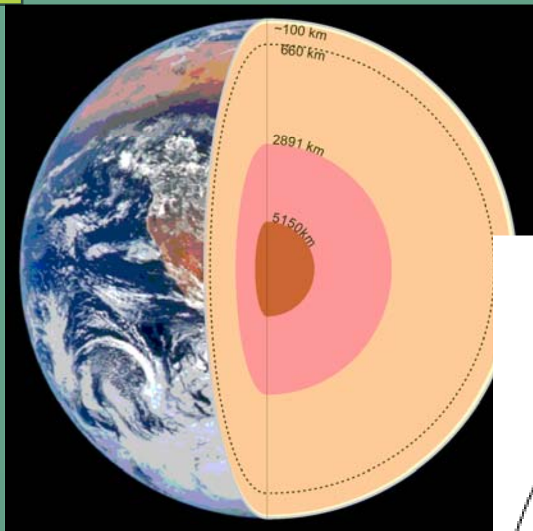


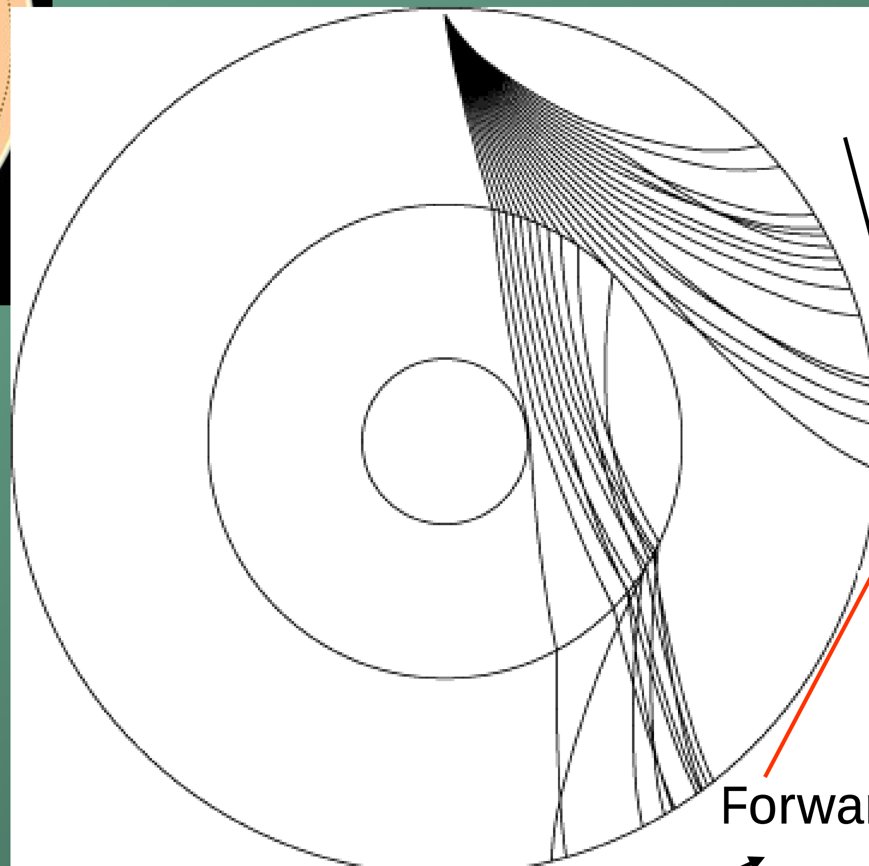


science for a changing world

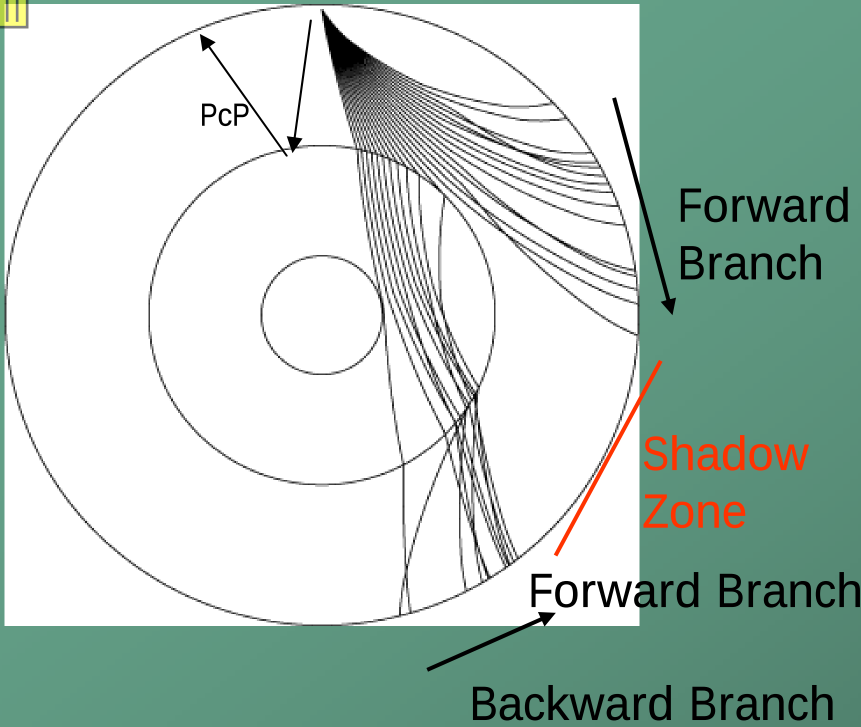

~100 km
660 km
2891 km
5150km

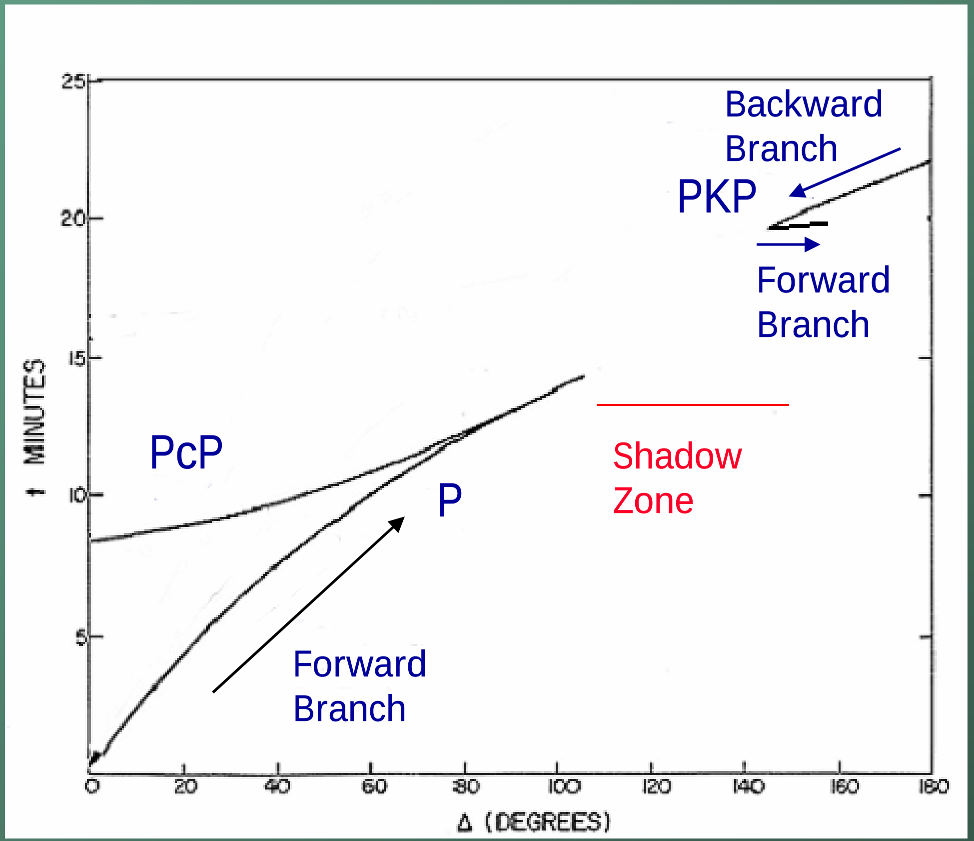

Forward Branch
Backward Branch





science for a changing world

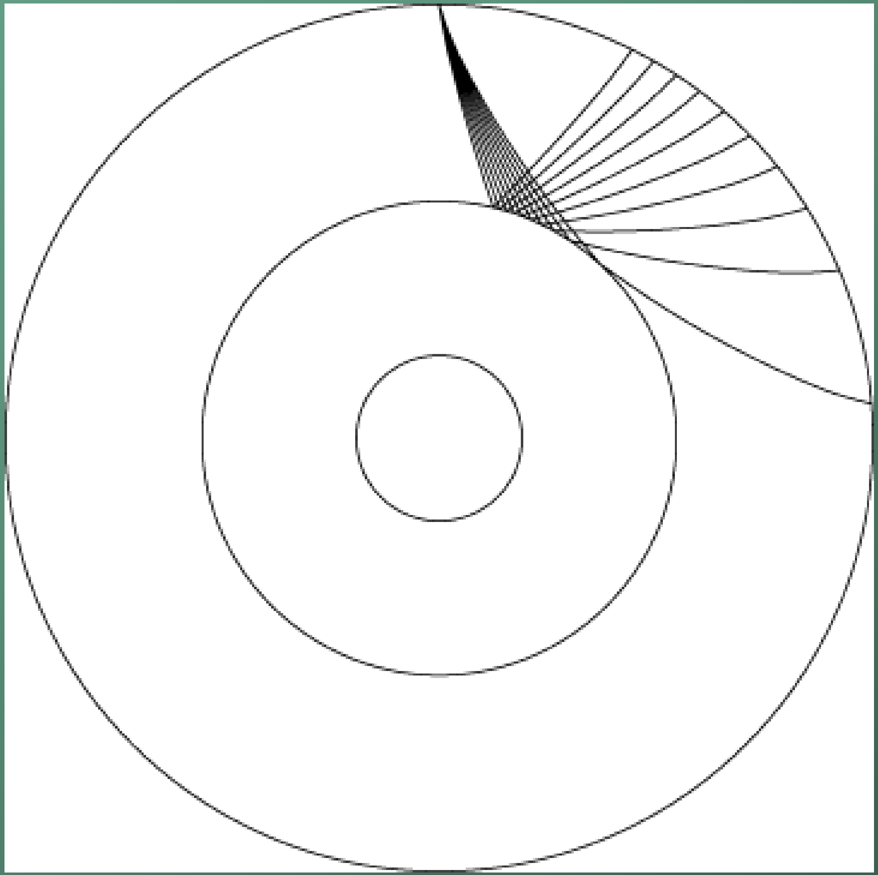




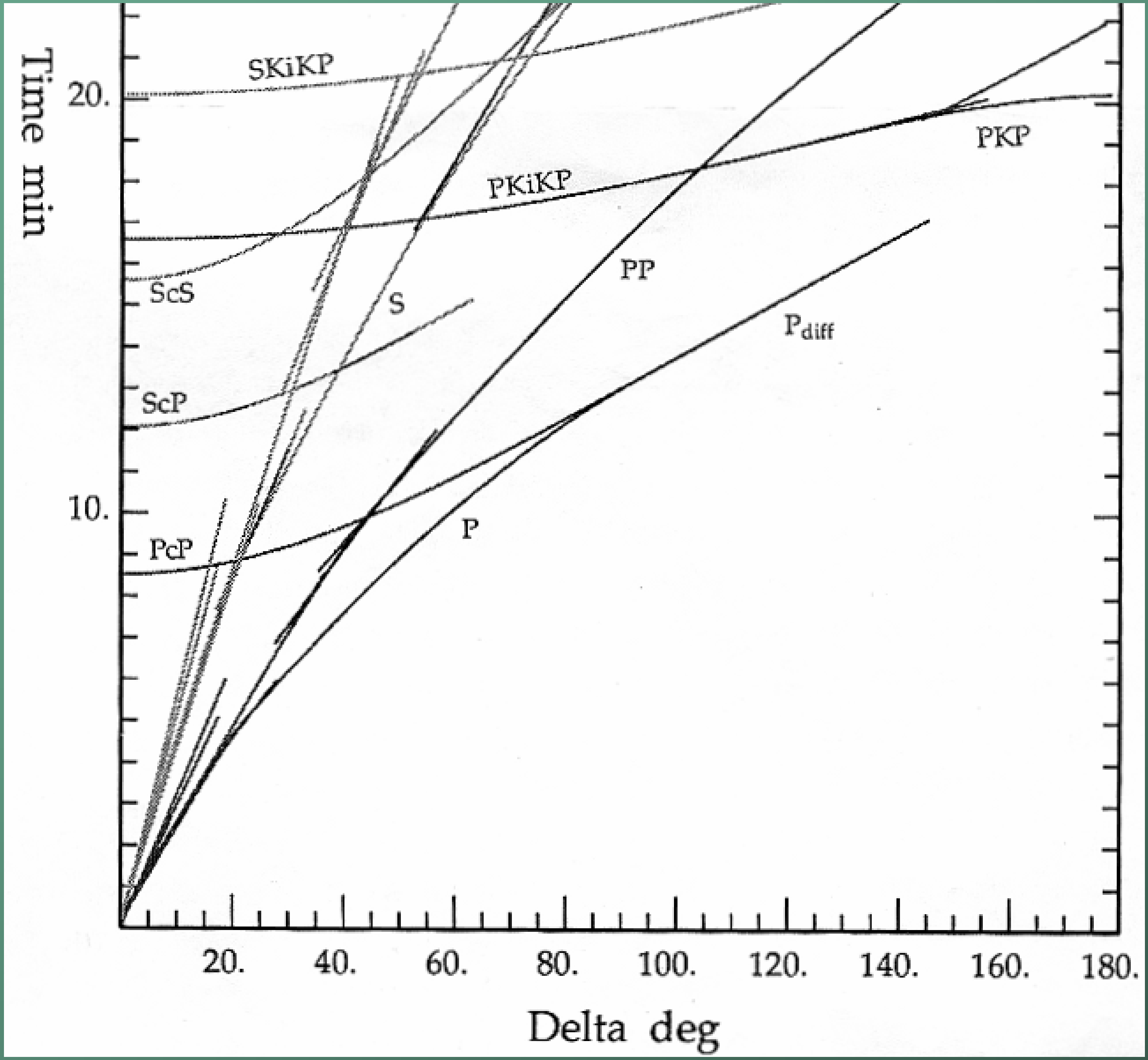
- 1912 Gutenberg observed shadow zone 105º to 143º
- 1939 Jeffreys fixed depth of core at 2898 km (using PcP)

USGS
science for a changing world

PcP



Core Reflections



P	**Mantle P**
S	**Mantle S**
K	**Outer core P**
I	**Inner core P**
c	**Reflection from the outer core**
i	**Reflection from the inner core**
diff	**Diffracted arrival**

IASP91, Kennett and Engdahl, 1991

USGS
science for a changing world

Seismic Waves

Aspects of Waves not Explained by Ray Theory

- Different types of waves (P, S)
- Surface Waves
- Static Displacements
- Frequency content

Wave Equation

$$\frac{\partial^2 u_1}{\partial x_1^2} = \frac{1}{c}\frac{\partial^2 u_1}{\partial t^2}$$

1-D wave equation

c = propagation speed

This is the equation that explains the waves on a spring: constant velocity wave propagation, no mass transfer, different from circulation eq.

USGS
science for a changing world

1-D Wave Equation

$$\frac{\partial^2 u_1}{\partial x_1^2} = \frac{1}{c}\frac{\partial^2 u_1}{\partial t^2}$$

Solution

$$u(x,t) = A\sin[\omega(t \pm x/c)]$$

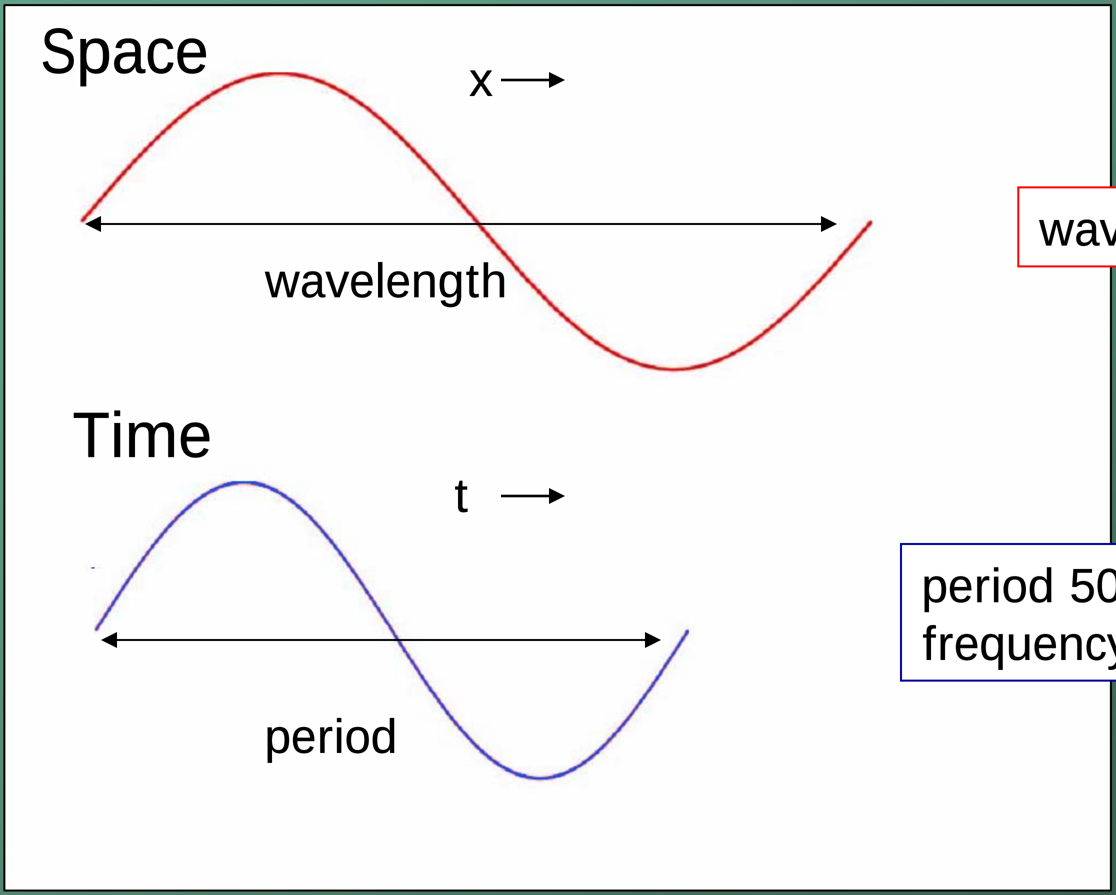
$$T = \frac{2\pi}{\omega}$$

T = wave period

ω = angular frequency

USGS
science for a changing world

Wave Period and Wavelength



Velocity 6 km/s

wavelength 300 km

period 50 s
frequency = 1/period= 0.02 hz

Velocity = Wavelength / Period

	Period	Wavelength
Body waves （P· S）	**0.01 to 50 sec**	**50 m to 500 km**
Surface waves	**10 to 350 sec**	**30 to 1000 km**
Free Oscillations	**350 to 3600 sec**	**1000 to 10000 km**
Static Displacements	∞	**-**

science for a changing world

3-D Wave Equation with Source

$$\rho \frac{\partial^2 u}{\partial t^2} = f + (\lambda + 2\mu)\nabla(\nabla \cdot u) - \mu\nabla \times (\nabla \times u)$$

source — spatial 2nd derivative

Near-field Terms (Static Displacements)

Solution

$$u(x,t) = \frac{1}{4\pi\rho} A^N \frac{1}{r^4}\int_{r/\alpha}^{r/\beta} \tau \cdot M_0(t-\tau)d\tau + \frac{1}{4\pi\rho\alpha^2} A^{IP} \frac{1}{r^2} M_0(t-\frac{r}{\alpha}) + \frac{1}{4\pi\rho\beta^2} A^{IS} \frac{1}{r^2} M_0(t-\frac{r}{\beta})$$

$$+ \frac{1}{4\pi\rho\alpha^3} A^{FP} \frac{1}{r} \dot{M}_0(t-\frac{r}{\alpha}) + \frac{1}{4\pi\rho\beta^3} A^{FS} \frac{1}{r} \dot{M}_0(t-\frac{r}{\beta})$$

Far-field Terms (P, S Waves)

USGS science for a changing world

Near-field terms

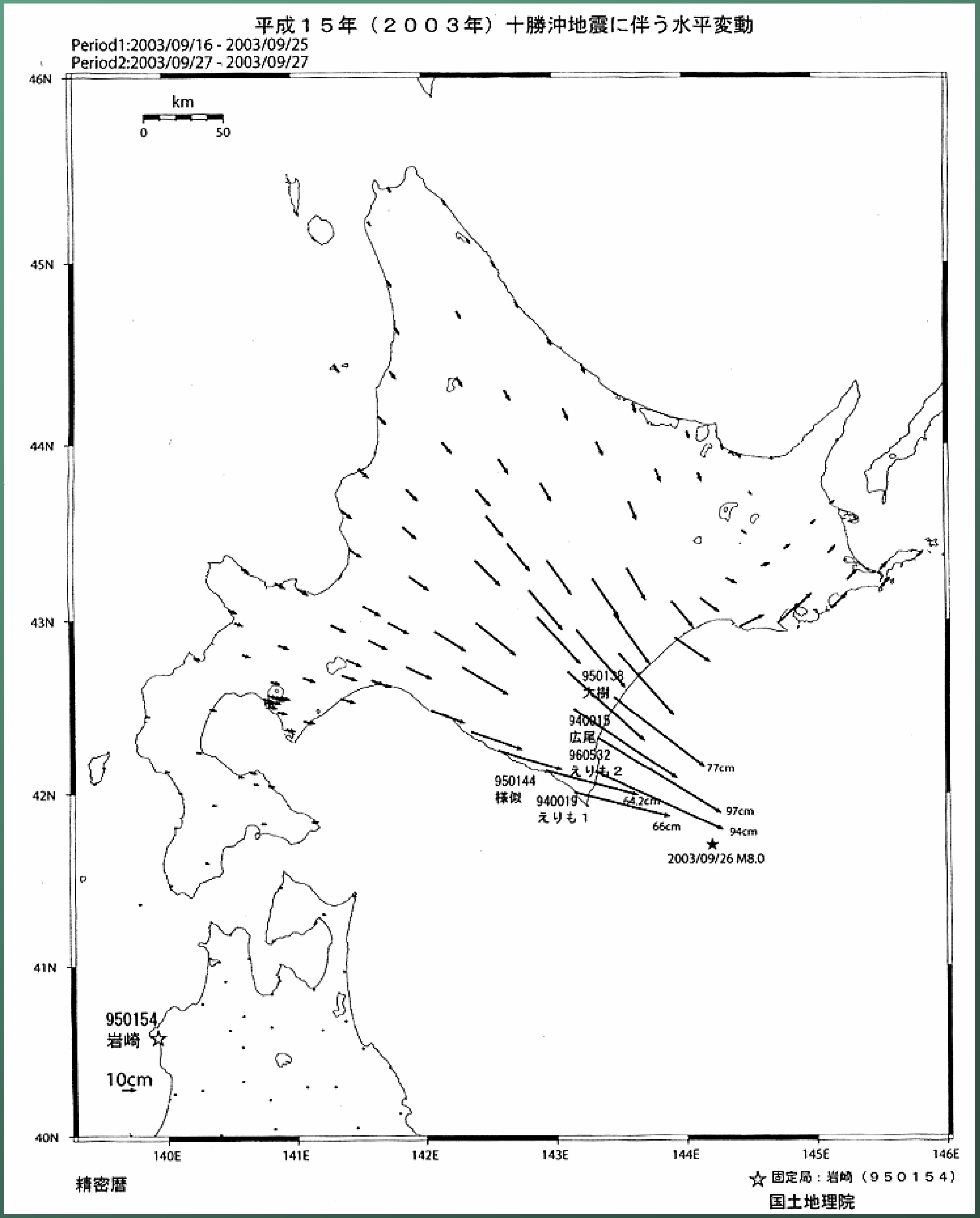
- Static displacements
- Only significant close to the fault
- Source of tsunamis

Bei-Fung Bridge near Fung-Yan city, 1999 Chi-Chi, Taiwan earthquake

USGS
science for a changing world

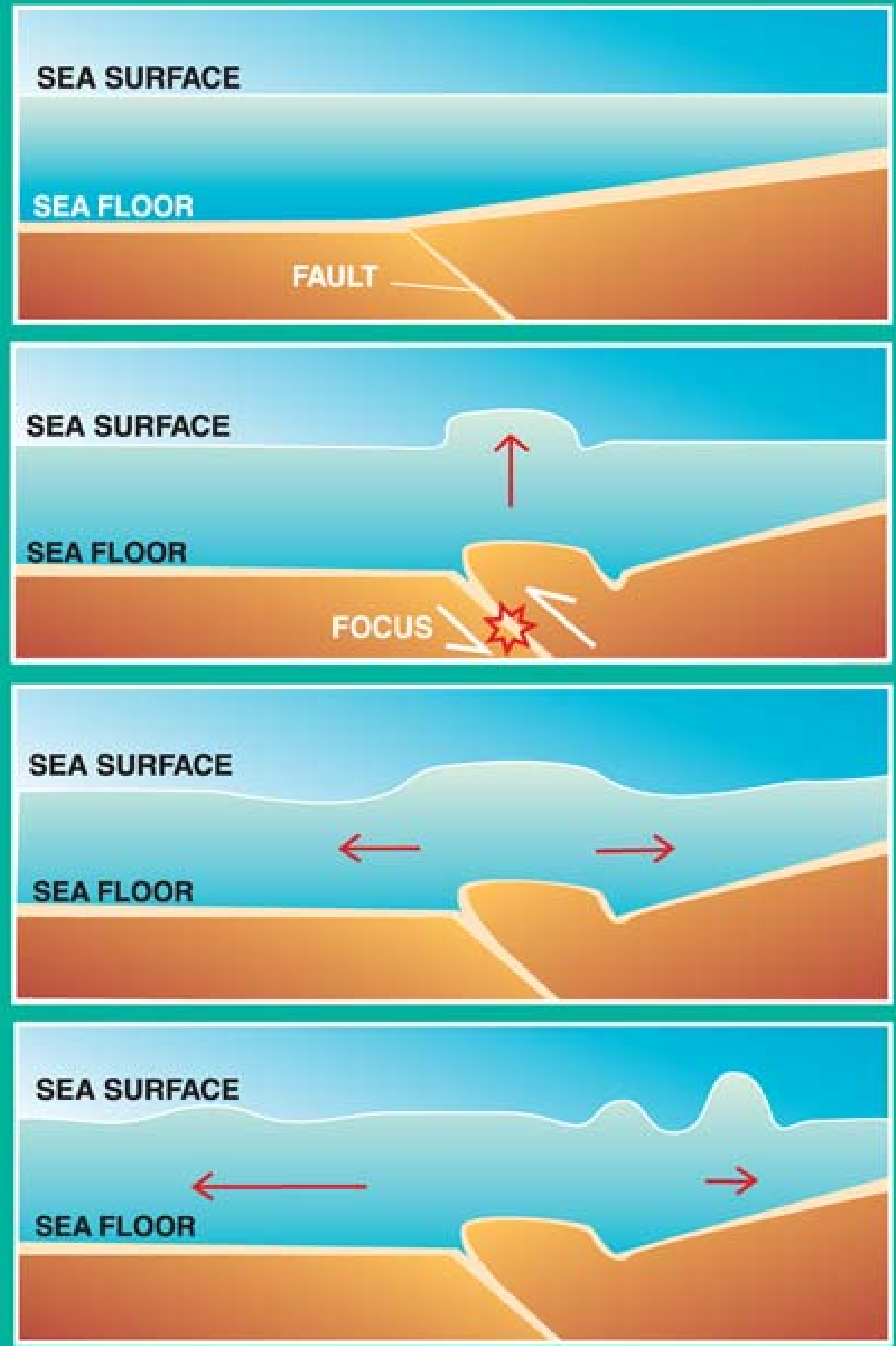
Static displacements

Co-seismic deformation of 2003 Tokachi-oki Earthquake (M8.0)



science for a changing world

Generation of Tsunami from Near-field Term

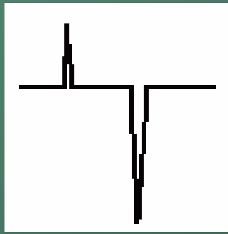


USGS
science for a changing world

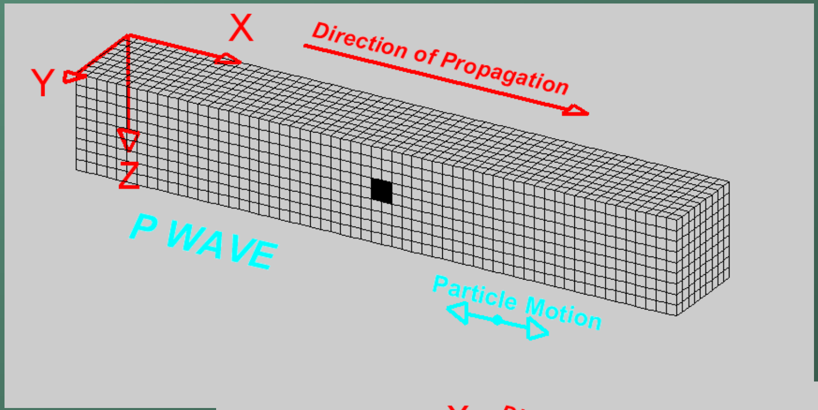
UNESCO-IOC
(Great waves)

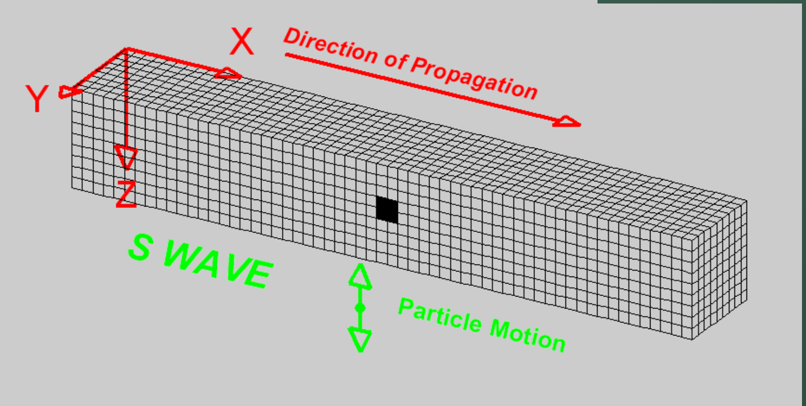
Far-field Terms

$$+\frac{1}{4\pi\rho\alpha^3}A^{FP}\frac{1}{r}\dot{M}_0(t-\frac{r}{\alpha})+\frac{1}{4\pi\rho\beta^3}A^{FS}\frac{1}{r}\dot{M}_0(t-\frac{r}{\beta})$$

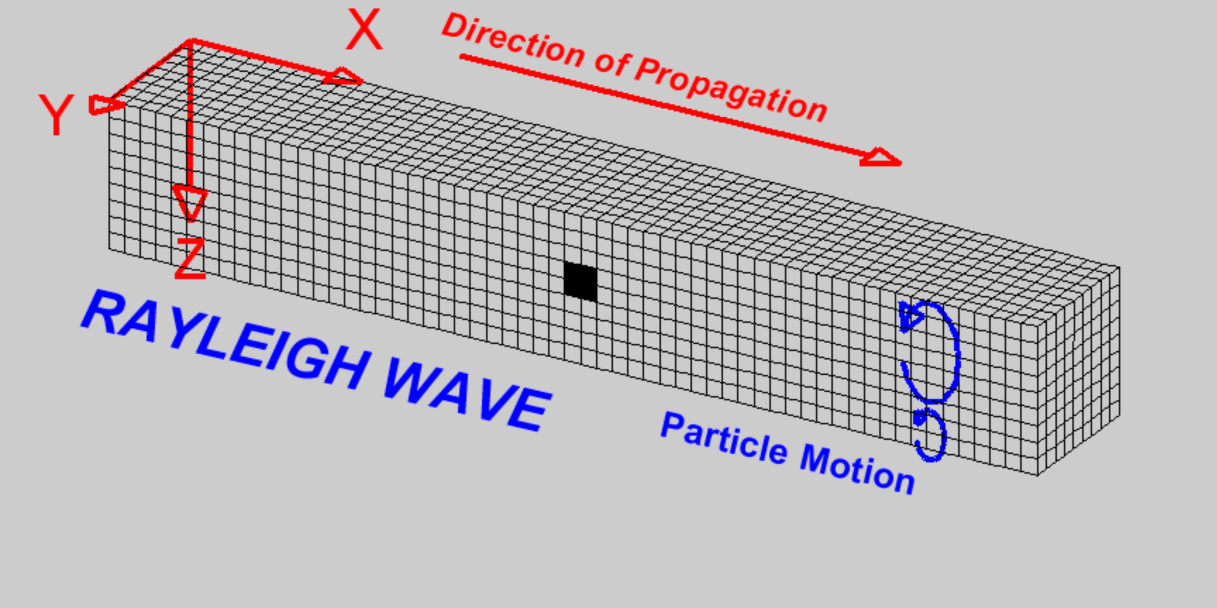


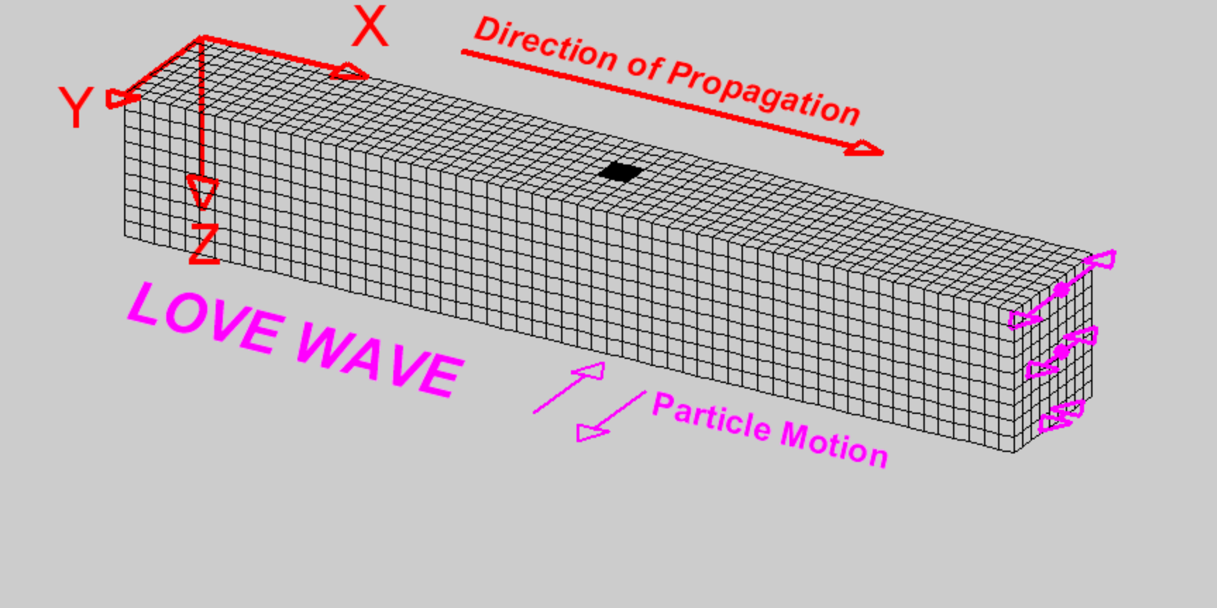
- Propagating Waves
- No net displacement
- P waves
- S waves





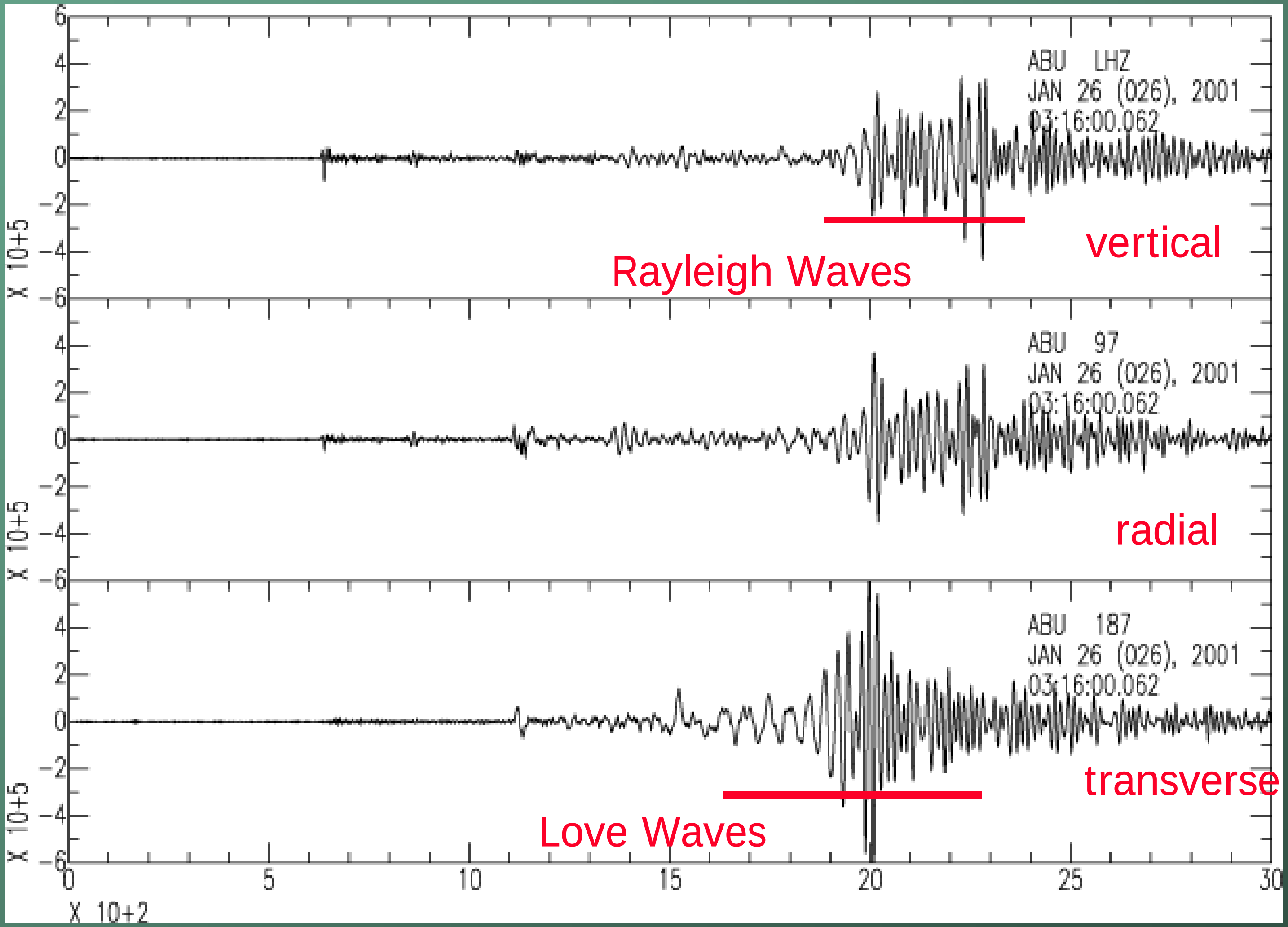
surface waves





USGS
science for a changing world

January 26, 2001 Gujarat, India Earthquake (Mw7.7)



Recorded in Japan at a distance of 57º (6300 km)

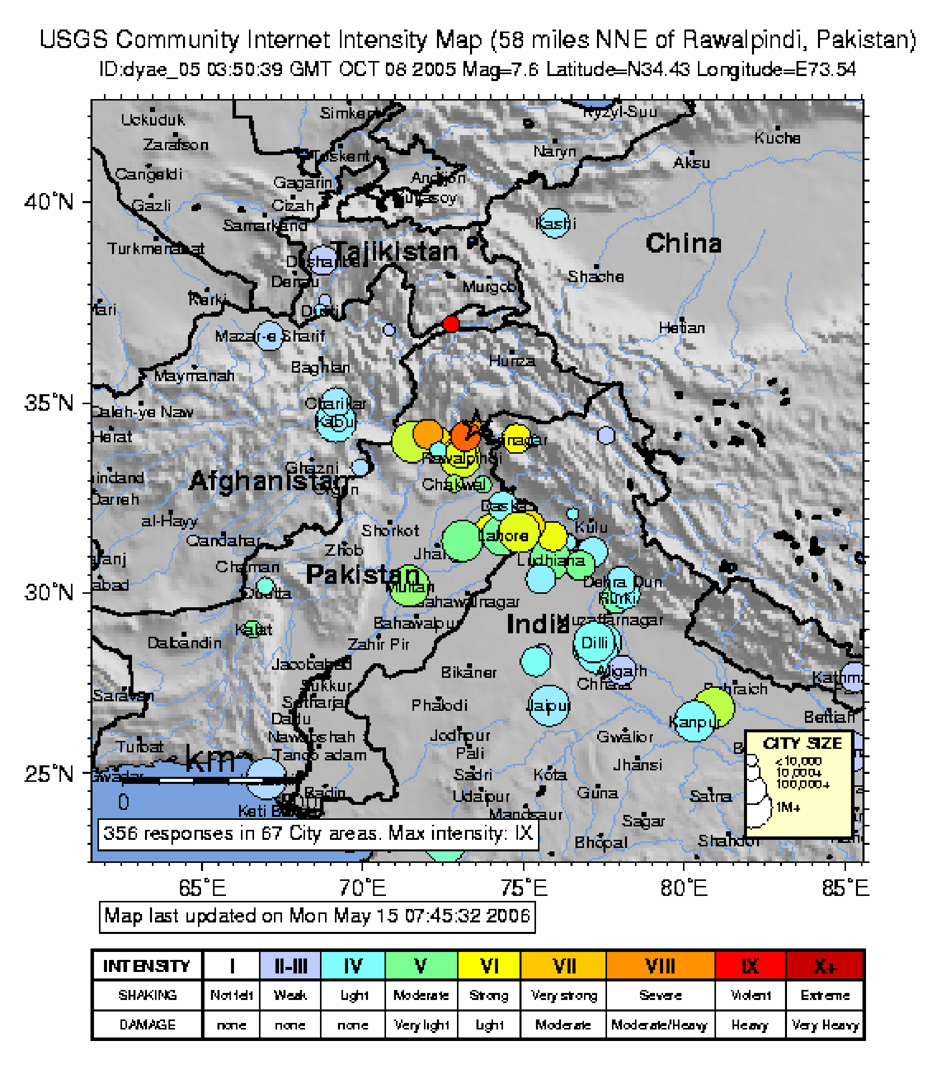
USGS
science for a changing world

Amplitude and Intensity

Seismic waves loose amplitude with distance traveled - attenuation

$$A(t) = A_0 e^{-\omega_0 t/2Q}$$

So the amplitude of the waves depends on distance from the earthquake. Therefore unlike magnitude intensity is not a single number.



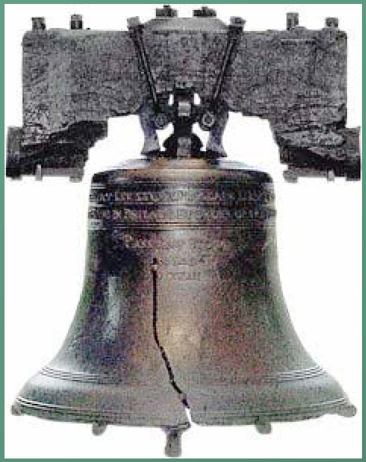
Modified Mercalli Intensity

I	Barely felt
II	Felt by only few people
III	Felt noticeably, standing autos rock slightly
IV	Felt by many, windows and walls creak
V	Felt by nearly everyone, some dished and windows broken
VI	Felt by all, damaged plaster and chimneys
VII	Damage to poorly constructed buildings
VIII	Collapse of poorly constructed buildings, slight damage to well built structures
IX	Considerable damage to well constructed buildings, buildings shifted off foundations
X	Damage to well built wooden structures, some masonry buildings destroyed, train rails bent, landslides
XI	Few masonry structure remain standing, bridges destroyed, ground fissures
XII	Damage total

USGS
science for a changing world

Normal Modes

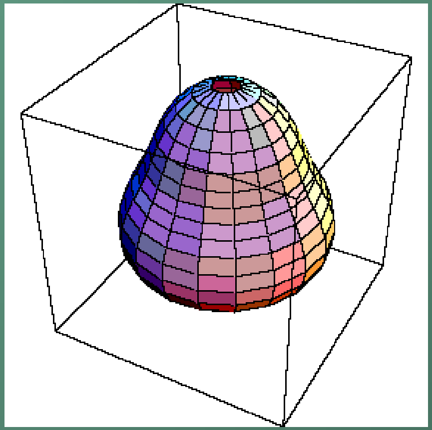
Liberty Bell (USA)



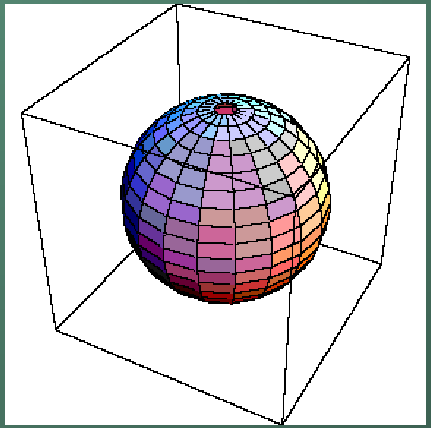
Useful for studies of

- Interior of the Earth
- Largest earthquakes

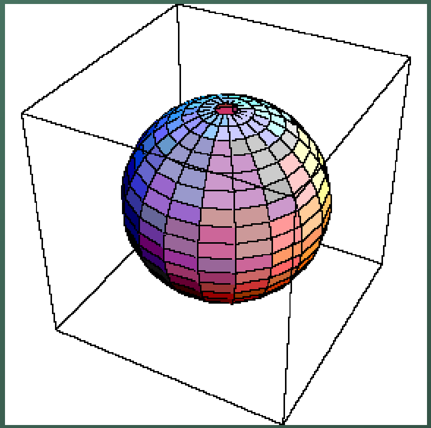
l=1 m=1



l=1 m=2

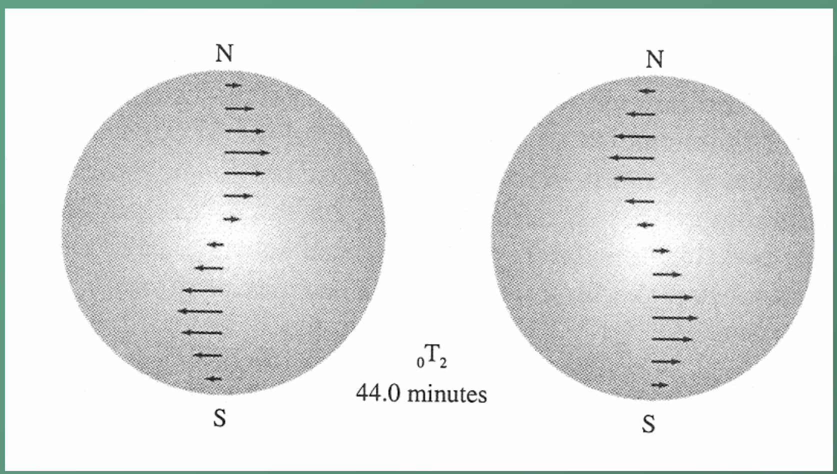


l=1 m=3

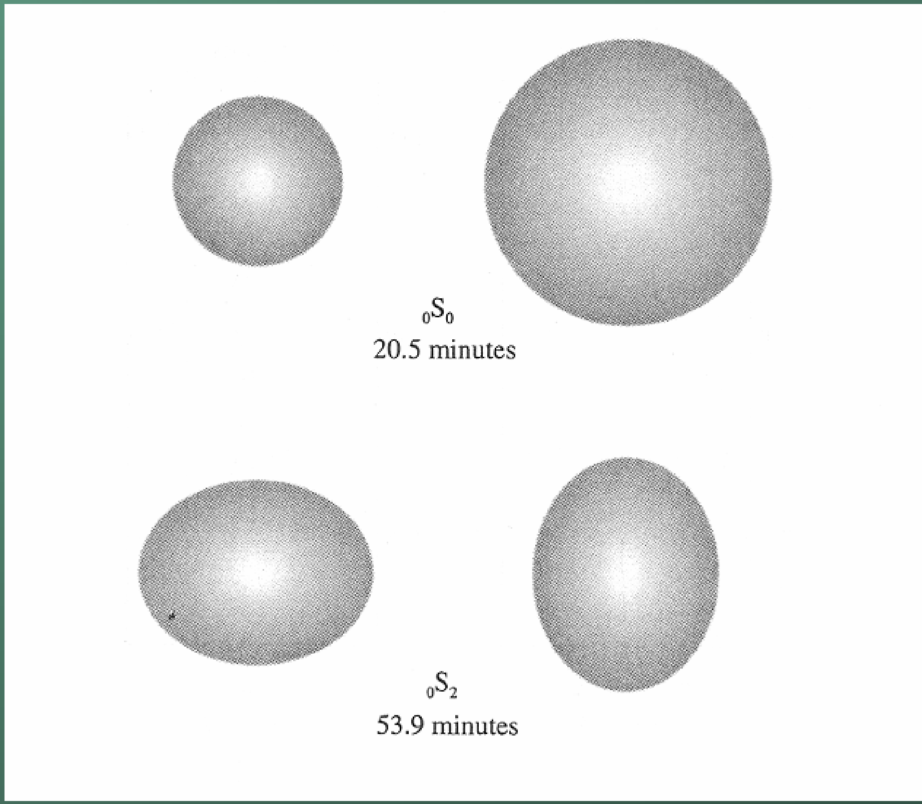


Houseman http://earth.leeds.ac.uk/~greg/?Sphar/index.html

Toroidal and Spheroidal Modes



Toroidal



Spheroidal

USGS
science for a changing world

Dahlen and Tromp Fig. 8.5, 8.17

Natural Vibrations of the Earth

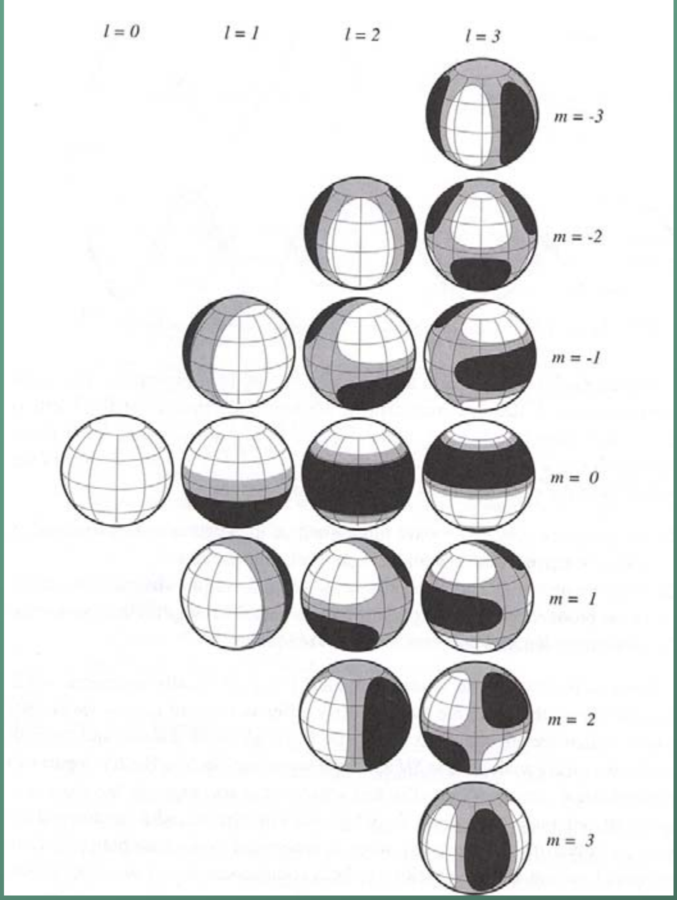


TABLE 4.2 Some Observed Normal-Mode Periods

Spheroidal modes	T(s)	Toroidal modes	T(s)
${}_0S_0$	1227.52	${}_0T_2$	2636.38
${}_0S_2$	3233.25	${}_0T_{10}$	618.97
${}_0S_{15}$	426.15	${}_0T_{20}$	360.03
${}_0S_{30}$	262.09	${}_0T_{30}$	257.76
${}_0S_{45}$	193.91	${}_0T_{40}$	200.95
${}_0S_{60}$	153.24	${}_0T_{50}$	164.70
${}_0S_{150}$	66.90	${}_0T_{60}$	139.46
${}_1S_2$	1470.85	${}_1T_2$	756.57
${}_1S_{10}$	465.46	${}_1T_{10}$	381.65
${}_2S_{10}$	415.92	${}_2T_{40}$	123.56

USGS
science for a changing world

Shearer Ch.8.6
Lay and Wallace, Ch. 4.6

Summary

Rays

Earth structure causes complicated ray paths through the Earth (P, PKP, PcP)

Wave theory explains

- P and S waves
- Static displacements
- Surface waves

Normal Modes

The Earth rings like a bell at long periods