Appendix E. Incorporating tides into probability calculations

Including tides in the probability calculation is essentially a source of aleatory uncertainty, because it is unknown at what tidal stage the tsunami wavetrain will arrive at Seaside (Mofjeld *et al.*, 1997; Mofjeld *et al.*, in press). First, consider the far-field case where only one rupture scenario (magnitude, slip, area) is considered per source region (j). For each tsunami amplitude A_j (corresponding to each j source), the tides produce a probability density function $f(y, A)$ that describes the distribution in height of the maximum waves, where y is the total wave height, including tsunami and tidal amplitudes. To get the exceedance rate for the jth source as a function of y, we integrate the corresponding probability density function (PDF) as follows:

$$F_j\left(y, A_j\right) \equiv \int_y^{\infty} f\left(y', A\right) dy'. \tag{E1}$$

To get an explicit form for $F(y, A)$, a Gaussian distribution (Mofjeld *et al.*, in press) is a reasonable approximation to the PDF:

$$f(y, A) = B \exp\left[-(y-\eta_0)^2 / 2\sigma^2\right], \; B^{-1} = \sqrt{2}\pi\sigma, \tag{E2}$$

where

$$\eta_0(A) = A + \mathrm{MSL} + C(\mathrm{MHHW} - \mathrm{MSL}) \exp\left[-\alpha\,(A/\sigma_0)^{\beta}\right] \tag{E3}$$

and

$$\sigma(A) = \sigma_0 - C'\sigma_0 \exp\left[-\alpha'\,(A/\sigma_0)^{\beta'}\right]. \tag{E4}$$

The parameters C, α, β, C', α', and β' are specific to the tides at a given location (see Appendix A). The integral $F(y, A)$ is then

$$F(y, A) = \frac{1}{2}\mathrm{erfc}\left[\left(y-\eta_0\right) \Big/ \sqrt{2}\sigma\right], \mathrm{erfc}(z) \equiv 1 - \mathrm{erf}(z). \tag{E5}$$

Here, $\mathrm{erf}(z) = \left(2 / \sqrt{\pi}\right) \int_0^z \exp\left(-t^2\right) dt$ is the standard form of the error function.

The rate at which $y' \geq y$ for a given source (λ_j) is the mean recurrence rate of that design earthquake (ν_j) multiplied by F_j:

$$\lambda_j\left(y' \geq y\right) = \nu_j F_j\left(y' \geq y\right). \tag{E6}$$

If for all source regions there is only one design earthquake per region, then the aggregate exceedance rate would be (since we are dealing with discrete regions):

$$\lambda\left(y' \geq y\right) = \sum_j \lambda_j\left(y' \geq y\right). \tag{E7}$$

The exceedance probability at Seaside, assuming that the arrival times are Poissonian, is

$$P\left(y' > y\right) = 1 - e^{-\lambda T}, \tag{E8}$$

where T is the exposure time of interest (1 year).

Next, consider just the local case where a distribution of tsunami amplitudes is obtained from multiple slip distributions available for a specified magnitude, average slip, area. If $p(A_{\text{local}})$ is the PDF for local tsunami amplitudes, then

$$E_{\text{local}}\left(y' \geq y\right) = \int_0^\infty p\left(A_{\text{local}}\right) F\left(y, A_{\text{local}}\right) dA. \tag{E9}$$

$p(A_{\text{local}})$ can be determined from a normalized histogram as in Mofjeld *et al.* (in press) or by approximation to a Gaussian distribution, similar to $f(y, A)$. Inclusion of multiple realizations for local rupture (aleatory uncertainty of the source) is discussed further in Section 5.2, "Source Specification," for Cascadia subduction zone earthquakes below. It follows that the local exceedance rate is given by

$$\lambda_{\text{local}}\left(y' \geq y\right) = \nu_{\text{local}} E_{\text{local}}\left(y' \geq y\right). \tag{E10}$$

Therefore, the "Grand Total" exceedance rate (local + far field) can be given by

$$\lambda\left(y' \geq y\right) = \lambda_{\text{local}}\left(y' \geq y\right) + \sum_{j}^{\text{far-field}} \lambda_j\left(y' \geq y\right). \tag{E11}$$

The objective, of course, is to find y corresponding to $\lambda = 0.01\ \text{yr}^{-1}$ and $\lambda = 0.002\ \text{yr}^{-1}$.

References

Mofjeld, H.O., M.G.G. Foreman, and A. Ruffman, 1997, West Coast tides during Cascadia subduction zone tsunamis. *Geophysical Research Letters, 24*, 2215–2218.

Mofjeld, H.O., F.I. González, V.V. Titov, A.J. Venturato, and J.C. Newman, in press, Effects of tides on maximum tsunami wave heights: Probability distributions. *Journal of Atmospheric and Oceanic Technology.*